Applied Mathematical Modelling 33 (2009) 3237–3244

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A generalized model for data envelopment analysis with interval data

G.R. Jahanshahloo, F. Hosseinzadeh Lotfi, M. Rostamy Malkhalifeh *, M. Ahadzadeh Namin

Islamic Azad University, Department of Mathematics, Science and Research Branch, Tehran 14515-775, Iran

ARTICLE INFO

Article history:
Received 23 January 2008
Received in revised form 18 October 2008
Accepted 22 October 2008
Available online 5 November 2008

Keywords:
Data envelopment analysis
FDH
GDEA
Interval data

ABSTRACT

Data envelopment analysis (DEA) is a method to estimate the relative efficiency of decision-making units (DMUs) performing similar tasks in a production system that consumes multiple inputs to produce multiple outputs. So far, a number of DEA models with interval data have been developed. The CCR model with interval data, the BCC model with interval data and the FDH model with interval data are well known as basic DEA models with interval data. In this study, we suggest a model with interval data called interval generalized DEA (IGDEA) model, which can treat the stated basic DEA models with interval data in a unified way. In addition, by establishing the theoretical properties of the relationships among the IGDEA model and those DEA models with interval data, we prove that the IGDEA model makes it possible to calculate the efficiency of DMUs incorporating various preference structures of decision makers.

1. Introduction

Data envelopment analysis (DEA) is a non-parametric technique for measuring and evaluating the relative efficiencies of a set of entities, called decision-making units (DMUs), with common inputs and outputs [1–4]. Examples include schools, hospitals, libraries and, more recently, the whole economic and social systems, in which outputs and inputs are always multiple in character.

In recent years, in different applications of DEA, inputs and outputs have been observed whose values are indefinite [5–7]. Such data are called "inaccurate". Inaccurate data can be probabilistic, interval, ordinal, qualitative, or Fuzzy. Therefore, some papers were presented on the theoretical development of DEA with interval data, of which we can name Despotis and Smirlis [8] and Jahanshahloo et al. [9].

Yun et al. [10] proposed a generalized model for DEA, called GDEA model, which can treat basic DEA models, specifically the CCR model, the BCC model and the FDH model in a unified way.

In this study, we propose a generalized model with interval data for interval DEA(IDEA), called IGDEA model, which can treat basic IDEA models, specifically the CCR, BCC, FDH models with interval data in a unified way. In addition, we show the theoretical properties of the relationships among the IGDEA and those IDEA models.

2. DEA model

We assume that there are **n** DMUs to be evaluated indexed by $j = 1, 2, \dots, n$. And each DMU is assumed to produce **s** different outputs from **m** different inputs. Let the observed input and output vectors of DMU_j be $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$ respectively, where all components of vectors X_j and Y_j for all DMUs are non-negative, and each DMU has at least one strictly positive input and output. For evaluating the efficiency of DMU_o we use some models as follows:

* Corresponding author. Tel./fax: +98 2144804172.
E-mail address: M.Rostamy_malkhalifeh@yahoo.com (M.R. Malkhalifeh).

0307-904X/$ - see front matter © 2008 Published by Elsevier Inc.
doi:10.1016/j.apm.2008.10.030

$$
\begin{aligned}
\text{Min} \quad & \theta - \varepsilon 1 S_z, \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_x = \theta x_{io}, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{ij} - s_y = y_{ro}, \quad r = 1, \ldots, s, \\
& \lambda_j \epsilon \Lambda, j = 1, 2, \ldots, n,
\end{aligned}
\tag{1}
$$

where $S_z = (s_x, s_y)$ and Λ are as follows:

$$
\begin{aligned}
& \Lambda = \Lambda_{CCR} = \{\lambda_j | \lambda_j \geqslant 0,\ j = 1, 2, \ldots, n\}, \\
& \Lambda = \Lambda_{BCC} = \left\{\lambda_j | \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geqslant 0, j = 1, 2, \ldots, n\right\}, \\
& \Lambda = \Lambda_{FDH} = \left\{\lambda_j | \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \epsilon \{0, 1\}, j = 1, 2, \ldots, n\right\},
\end{aligned}
$$

Definition 1. DMU_o is $CCR - efficient$ if and only if $\theta^* = 1$ and $S_z^* = 0$ for Λ equal to ΛCCR. BCC efficiency and FDH efficiency are defined likewise.

3. Interval DEA

Let the input and output values of any DMU_o be located in a certain interval, where x_{ij}^l and x_{ij}^u are the lower and upper bounds of the jth DMU, respectively, and y_{rj}^l and y_{rj}^u are the lower and upper bounds of the rth DMU, respectively; that is to say, $x_{ij}^l \leqslant x_{ij} \leqslant x_{ij}^u$ and $y_{rj}^l \leqslant y_{rj} \leqslant y_{rj}^u$.

Such data are called interval data, because they are located in intervals. Note that always $x_{ij}^l \leqslant x_{ij}^u$ and $y_{rj}^l \leqslant y_{rj}^u$. If $x_{ij}^l = x_{ij}^u$, then the ith input of the jth DMU has a definite value.

The BCC model for evaluating DMU_o with interval data is as follows:

$$
\begin{aligned}
\text{Max} \quad & U^t Y_o + u_o, \\
\text{s.t.} \quad & U^t Y_j - V^t X_j + u_o \leqslant 0 \quad j = 1, 2, \ldots, n, \\
& V^t X_o = 1, \\
& U \geqslant 1\varepsilon, V \geqslant 1\varepsilon,
\end{aligned}
\tag{2}
$$

where $Y_j = [y_{1j}, y_{2j}, \ldots, y_{sj}]$, $X_j = [x_{1j}, x_{2j}, \ldots, x_{mj}]$, $Y_o = [y_{1o}, y_{2o}, \ldots, y_{so}]$ and $X_o = [x_{1o}, x_{2o}, \ldots, x_{mo}]$ for each $j = 1, 2, \ldots, n$.

In problem (2), one can see that all parameters of the problem are in intervals and the efficiency of DMU_o is also located in an interval. The upper and lower bounds of the relative efficiency of DMU_o are obtained by solving the following problems, respectively.

$$
\begin{aligned}
\bar{\theta} = \text{Max} \quad & U^t Y_o^u + u_o, \\
\text{s.t.} \quad & U^t Y_j^l - V^t X_j^u + u_o \leqslant 0 \quad j = 1, 2, \ldots, n,\ j \neq o, \\
& U^t Y_o^u - V^t X_o^l + u_o \leqslant 0, \\
& V^t X_o^l = 1, \\
& U \geqslant 1\varepsilon, \quad V \geqslant 1\varepsilon,
\end{aligned}
\tag{3}
$$

and

$$
\begin{aligned}
\underline{\theta} = \text{Max} \quad & U^t Y_o^l + u_o, \\
\text{s.t.} \quad & U^t Y_j^u - V^t X_j^l + u_o \leqslant 0 \quad j = 1, 2, \ldots, n,\ j \neq o, \\
& U^t Y_o^l - V^t X_o^u + u_o \leqslant 0, \\
& V^t X_o^u = 1, \\
& U \geqslant 1\varepsilon, \quad V \geqslant 1\varepsilon,
\end{aligned}
\tag{4}
$$

If we suppose $u_o = 0$ in models (2)–(4), we have the CCR models.

Considering that the efficiency of any DMU lies in an interval, all DMUs can be divided into one of the three following classes: