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Analytical solution for electromagnetothermoelastic behaviors of a functionally graded piezoelectric hollow cylinder

Hong-Liang Dai *, Li Hong, Yi-Ming Fu, Xia Xiao

College of Mechanics and Aerospace, Hunan University, Changsha 410082, Hunan Province, People's Republic of China

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A B S T R A C T

Analytical study for electromagnetothermoelastic behaviors of a hollow cylinder composed of functionally graded piezoelectric material (FGPM), placed in a uniform magnetic field, subjected to electric, thermal and mechanical loads are presented. For the case that the electric, magnetic, thermal and mechanical properties of the material obey an identical power law in the radial direction, exact solutions for electric displacement, stresses, electric potential and perturbation of magnetic field vector in the FGPM hollow cylinder are determined by using the infinitesimal theory of electromagnetothermoelasticity. Some useful discussions and numerical examples are presented to show the significant influence of material inhomogeneity, and adopting a certain value of the inhomogeneity parameter β and applying suitable electric, thermal and mechanical loads can optimize the FGPM hollow cylindrical structures. This will be of particular importance in modern engineering design.

1. Introduction

Functionally graded piezoelectric material (FGPM) is a kind of piezoelectric material with material composition and properties varying continuously along certain directions. FGPM is the composite material intentionally designed so that they possess desirable properties for some specific applications. The advantage of this new kind of materials can improve the reliability of life span of piezoelectric devices. Recently, there has been growing interest in materials deliberately fabricated so that their electric, magnetic, thermal and mechanical properties vary continuously in space on the macroscopic scale. This research subject is so new that only a few results can be found in the literatures. Previous studies on the subject considered FGPM including those, for example, by Wu et al. [1], Chen et al. [2], Wu et al. [3], Chen et al. [4], Pan and Han [5], Hu et al. [6], Sladek et al. [7–9], where additional references can be found. Lim and He [10] obtained an exact solution of a compositionally graded piezoelectric layer under uniform stretch, bending and twisting. By means of singular integral equation technique, Wang [11] investigated the mode III crack problems in functionally gradient piezoelectric materials. An exact three-dimensional analysis was presented by Zhong and Shang [12] for a functionally gradient piezoelectric rectangular plate that is simply supported and grounded along its four edges. By means of an analytical–numerical method, Han and Liu [13] studied elastic waves in a functionally graded piezoelectric cylinder. Utilizing the Fourier transform technique, Ueda [14] investigated thermally induced fracture of a functionally graded piezoelectric layer. Sun et al. [15] investigated the behavior of a crack in functionally graded piezoelectric/piezo-magnetic materials subjected to an anti-plane shear loading. Lu et al. [16] derived exact solutions of a simply supported functionally graded piezoelectric plate/laminate under cylindrical bending. Ma et al. [17] investigated the electroelastic behavior of a Griffith crack in a functionally graded piezoelectric strip. Based

* Corresponding author. Tel./fax: +86 0731 8822330.
E-mail address: hldai520@sina.com (H.-L. Dai).

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Nomenclature

$\vec{U}, u$	displacement vector and radial displacement (m)
r	radial variable (m)
a,b	inner and outer radii of the FGPM hollow cylinder (m)
$\sigma_i(i = r,\theta,z)$	components of stresses (N/m^2)
$T(r)$	temperature distribution (K)
$\phi(r)$	electric potential (W/A)
D_r	radial electric displacement (C/m^2)
$c_{ij}(i = 1,2; j = 1,2,3)$	elastic constant (N/m^2)
$e_{1i}(i = 1,2,3)$	piezoelectric constants (C/m^2)
g_{11}	dielectric constant (C^2 N m^2)
p_{11}	pyroelectric coefficient (C/m^2 K)
α_i,λ_i $(i = 1,2,3)$	thermal constants (1/K) and thermal modulus (N/m^2 K)
$\underline{J}$	electric current density vector
h, h_z	perturbation of magnetic field vector
μ_0	magnetic permeability (H/m)
$\underline{e}$	perturbation of electric field vector
H, H_z	magnetic intensity vector
f_z	Lorentz's force (kg/m^2 s^2)
k	thermal conduction coefficient (W/mK)
h	ratio of the convective heat-transfer coefficient (W/K)
T_0	temperature (K)

Non-dimensional quantities
$R = \frac{r-a}{b-a}, \quad R1 = \frac{r}{a}, \quad T^* = \frac{T(r)}{T_0}, \quad \sigma_i^* = \frac{\sigma_i}{P_a}(i = r, \theta, z), \quad \phi^* = \frac{\phi}{\phi_a}, \quad h_z^* = \frac{h_z}{H_z}$

on the principles of linear thermopiezoelectricity, a layer-wise finite element formulation was developed by Lee [18] for piezoelectric materials was used to investigated the displacement and stress response of a functionally graded piezoelectric bimorph actuator. Utilizing the methods of Laplace and finite sine transformations, Ootao and Tanigawa [19,20] studied the transient piezothermoelastic problem of a functionally graded thermopiezoelectric strip and hollow sphere due to uniform heat supply. Huang and Shen [21] dealt with the nonlinear vibration and dynamic response of a functionally graded material plate with surface-bonded piezoelectric layers in thermal environments. Dai et al. [22] investigated electromagnetoelastic solutions for functionally graded piezoelectric solid cylinder and sphere. However, investigations on the exact electromagnetothermoelastic solution for FGPM hollow cylinder placed in a uniform magnetic field have not been found in the literatures.

The present paper is, upon employing simplifying assumptions, to present to an analytical solution of a FGPM hollow cylinder. The electric displacement, stresses, electric potential and perturbation of magnetic field vector distributions in the FGPM hollow cylinder with internal radius a and external radius b will be calculated. All material constants are assumed to the same power-law dependence through the wall thickness of the FGPM hollow cylinder, i.e.

$$\begin{aligned} &c_{ij}(r) = c_{ij}^0 r^\beta (i = 1,2; j = 1,2,3), \quad e_{1i}(r) = e_{1i}^0 r^\beta (i = 1,2,3), \quad g_{11}(r) = g_{11}^0 r^\beta, \\ &p_{11}(r) = p_{11}^0 r^\beta, \quad \alpha_i(r) = \alpha_i^0 r^\beta (i = 1,2,3), \quad \mu(r) = \mu_0 r^\beta. \end{aligned} \tag{1}$$

Here, subscript zero denotes corresponding value at the outer surface ($r = b$) of the FGPM hollow cylinder, and β is the inhomogeneous constant determined empirically. The range $-2 \leqslant \beta \leqslant 2$ to be used in the present study covers all the values of coordinate exponent encountered in the references cited earlier [12,23]. However, these values for β do not necessarily represent a certain material, various β values are used to demonstrate the effect of inhomogeneity on the electric displacement, stresses, electric potential and perturbation of magnetic field vector distributions.

2. Basic formulations of the problem

2.1. Derivation of equations

A long, FGPM hollow cylinder with perfect conductivity placed in a uniform magnetic field $\vec{H}(0, 0, H_z)$. For the axisymmetry plain strain assumption problem, the components of displacement, stresses, electric displacement and electric potential in the cylindrical coordinate (r,θ,z) are, respectively, expressed as $u(r)$, $\sigma_i(i = r,\theta,z)$, D_r and $\phi(r)$. The constitutive relations for the FGPM hollow cylinder subjected to a rapid change in temperature $T(r)$ are expressed as