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Applied Mathematical Modelling 31 (2007) 1282–1292

APPLIED MATHEMATICAL MODELLING

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Batch arrival queues under vacation policies with server breakdowns and startup/closedown times

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Received 1 September 2004; received in revised form 1 November 2005; accepted 20 February 2006
Available online 14 June 2006

Abstract

This paper studies the operating characteristics of an $M^{[x]}/G/1$ queueing system under vacation policies with startup/closedown times, where the vacation time, the startup time, and the closedown time are generally distributed. When all the customers are served in the system exhaustively, the server shuts down (deactivates) by a closedown time. After shutdown, the server operates one of (1) multiple vacation policy and (2) single vacation policy. When the server reactivates since shutdown, he needs a startup time before providing the service. If a customer arrives during a closedown time, the service is immediately started without a startup time. The server may break down according to a Poisson process while working and his repair time has a general distribution. We analyze the system characteristics for the vacation models.

Keywords: Closedown; Multiple vacation; Startup; Un-reliable; Single vacation

1. Introduction

This paper concentrates on an $M^{[x]}/G/1$ vacation system with server breakdown and startup/closedown times, reflecting more practical situations, that an un-reliable server turns off his service by a closedown time and a startup time is required before starting his service. An un-reliable server means that the server is typically subject to unpredictable breakdowns.

Queueing systems with server vacations have been attracted much attention to numerous researchers since Levy and Yechiali [1]. One of excellent surveys of queueing systems with server vacations can be referred to Doshi [2] and Takagi [3], which includes some applications. Gelenbe and Mitrani [4] used vacation queueing models to analyze some real life systems such as digital communications, computer network. Choudhury [5] modelled a batch arrival $M^{[x]}/G/1$ queueing system with a single vacation policy which extended the results of [2,3]. Batch arrival $M^{[x]}/G/1$ queueing systems with multiple vacations were first studied by Baba [6]. The variations and extensions of these models can be referred to Rosenberg and Yechiali [7], Lee et al. [8,9], Choudhury [10], Shomrony and Yechiali [11], Tang and Tang [12,13], Tang et al. [14], and many others. Recently, Ke

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doi:10.1016/j.apm.2006.02.010

[15] analyzed the optimal policy for M/G/1 queueing system with different vacation types and a startup time. Yechiali [16] examined an $M^{[x]}/G/1$ queueing system with a waiting server and vacations, in which the server, upon finding an empty system at the end of a vacation, activates a timer of duration T and waits dormant. Meanwhile the server operates the following policy: if a batch arrives during the dormant period T and then a new busy period starts, but if no arrivals occur, the server waits no more and takes another vacation.

Doshi [2] and Takagi [3] respectively examined GI/G/1 and M/G/1 queueing systems where the server requires a startup time before starting each of his service periods. Concerning queueing models with server breakdowns, Gaver [17] first proposed an ordinary M/G/1 queueing system with interrupted service and priorities. Gaver's system was extended to GI/G/1 case by Sengupta [18]. Li et al. [19] and Tang [20] investigated the behavior of the un-reliable server, and the effect of server breakdowns and repairs in the M/G/1 queueing models from both the queueing and reliability points of view. The concept of closedown time was first introduced by Takagi [3]. Niu and Takahashi [21] studied the performance analysis of the switched virtual connection (SVC) by a closedown time which is corresponding to an inactive timer during which the SVC resource is reserved to anticipate more customers (packets) from the same IP flow.

1.1. General description of models

In this paper, we consider an $M^{[x]}/G/1$ queueing system where an un-reliable server operates different policies with startup/closedown times. The detailed description of the models are given as follows:

1.1.1. Assumptions of the Model 1

1. Customers arrive in batches to occur according to a compound Poisson with rate λ. Let X_k denote the number of customers belonging to the kth arrival batch, where X_k, $k = 1, 2, 3, \ldots$, are with a common distribution $Pr[X_k = n] = \chi_n$, $n = 1, 2, 3, \ldots$.
2. Arriving customers at the system form a single waiting line and are served in the order of their arrivals. The server can serve only one customer at a time. The service time provided by a single server is an independent and identically distributed random variable (S) with a general distribution function $S(t)$.
3. The server is subject to breakdowns at any time with a Poisson breakdown rate α when he is working. Whenever the server fails, he is immediately repaired at a repair facility, where the repair time is an independent and identically distributed random variable (R) with a general distribution function $R(t)$.
4. A customer who arrives and finds the server busy or broken down must wait in the queue until a server is available. Although no service occurs during the repair period of a broken server, customers continue to arrive according to a Poisson process. In case the server breaks down when serving customers, he is sent for repair and the customer who has just being served should wait for the server back to complete his remaining service. Immediately after the server is fixed, he starts to serve customers until the system is empty, and the service time is cumulative.
5. Whenever the system becomes empty, the server shuts down (deactivates) by a closedown time D and then leaves for a vacation of random length V. If the server returns from a vacation to find no customers waiting in the queue, he immediately begins another vacation, and continues in this manner until he finds at least one customer waiting upon returning from a vacation. If the server returns from a vacation to find the system not empty, he immediately reactivates and performs a startup time with random length U before starting his continuous service. When the server completes his startup, he starts serving the waiting customers until the system becomes empty.
6. If a customer arrives during a closedown time, the service is immediately started without a startup time.

1.1.2. Assumptions of the Model 2

The first four and the last assumptions are the same as those in *Model 1*. However the fifth assumption now is that the server takes exactly one vacation when the system is empty. If the server returns from the vacation to find no customers waiting in the queue, he waits for the first batch to arrive while keeping the system idle. The server requires a startup time U before starting his each service period.