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ScienceDirect
Applied Mathematical Modelling 31 (2007) 1338–1350
APPLIED MATHEMATICAL MODELLING
www.elsevier.com/locate/apm

Impact of boundary conditions on entrainment and transport in gravity currents

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Received 1 March 2005; received in revised form 1 November 2005; accepted 30 March 2006
Available online 27 June 2006

Dedicated to Professor Guo Youzhong on the occasion of his 70th birthday

Abstract

Gravity currents have been studied numerically and experimentally both in the laboratory and in the ocean. The question of appropriate boundary conditions is still challenging for most complex flows. Gravity currents make no exception—appropriate, physically and mathematically sound boundary conditions are yet to be found. This task is further complicated by the technical limitations imposed by the current oceanographic techniques.

In this paper, we make a first step toward a better understanding of the impact of boundary conditions on gravity currents. Specifically, we use direct numerical simulations to investigate the effect that the popular Neumann, and less popular Dirichlet boundary conditions on the bottom continental shelf have on the entrainment and transport of gravity currents.

The finding is that gravity currents under these two different boundary conditions differ most in the way they transport heat from the top towards the bottom. This major difference occurs at medium temperature ranges. Entrainment and transport at high temperatures also show significant differences.

Keywords: Gravity currents; Entrainment; Transport; Boundary conditions; Boussinesq approximation; Energy equation

1. Introduction

A gravity or density current is the flow of one fluid within another caused by the temperature (density) difference between the fluids [1]. Gravity currents flowing down a sloping bottom that occur in the oceans (e.g.,

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0307-904X/$ - see front matter Published by Elsevier Inc.
doi:10.1016/j.apm.2006.03.033

those that flow down the Strait of Gibraltar and the Denmark Strait) are an important component of the thermohaline circulation [2], which in turn is important in climate and weather studies.

Oceanic gravity currents have been studied in the past, both in the laboratory and in the ocean. They have been modelled in various ways, starting from the streamtube models, e.g., [3,4] to more recent non-hydrostatic Boussinesq equations [5–7].

The question of appropriate boundary conditions for complex flows, such as the gravity currents we consider in this paper, is still challenging. This paper is a first step in an effort to bridge the gap between observed gravity currents and the assumptions made in modeling them. Using ocean observations to develop realistic boundary conditions for gravity currents is limited by the available technological means. Thus, the current boundary conditions used in the numerical simulation of gravity currents are based on physical intuition and mathematical simplicity and soundness.

The behavior of gravity currents under two different types of boundary conditions is studied in this paper. Specifically, the differences between gravity currents flowing with Neumann (insulation) and Dirichlet (fixed-temperature) bottom boundary conditions are investigated via direct numerical simulations. Quite possibly, other boundary conditions, such as Robin, would be more appropriate. However, given the popularity of Neumann [5,8–11] and, to a lesser extent, Dirichlet boundary conditions in numerical studies [12], we decided to focus first on these two types of boundary conditions.

Bottom Neumann boundary conditions for temperature may be assumed when the material on the continental shelf is a bad thermal-conductor (i.e., the current flows over "insulated" rock). Dirichlet boundary conditions may be assumed when the material on the continental shelf is a good thermal-conductor, making temperature nearly constant [13].

Dirichlet boundary conditions would be appropriate for the initial transient development of gravity currents. For instance, it is known that the Red Sea overflow shuts off in the summer [14]. Once this gravity current starts again, one could expect the temperature difference between the bottom layer and the overflow to have a transient impact on the mixing near the bottom. Also, such a temperature gradient could significantly affect the initial neutral buoyancy level, which is of ultimate interest for large-scale ocean and climate studies. Özgökmen et al. [15] found in numerical simulations that the neutral buoyancy level did not differ significantly from an analytical estimate, which does not account for mixing, because the bottom layer properties were isolated from vigorous mixing between the gravity current and the ambient fluid, and yet determined the neutral buoyancy level. Thus, any mechanism that can affect the properties of near bottom fluid is likely to change the neutral buoyancy level, at least during the initial stages of the development. This idea will be explored in a future study.

The rest of the paper is organized as follows: Section 2 presents the mathematical model used in our numerical study. Section 3 presents the numerical model used and the model configuration and parameters. Section 4 presents the velocity and temperature boundary and initial conditions used in the numerical simulation. Section 5 presents the numerical investigation of the effect of Neumann and Dirichlet boundary conditions on the bottom continental shelf on entrainment and transport in gravity currents. Five different metrics are used to this end. Finally, Section 6 presents a summary of our findings and directions of future research.

2. Mathematical setting

Consider a two-dimensional gravity current flowing downstream, with x as the horizontal (downstream) direction, and z the vertical direction (Fig. 1).

The momentum and continuity equations subject to the Boussinesq approximation can be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu_\mathrm{h}\frac{\partial^2 u}{\partial x^2} + \nu_\mathrm{v}\frac{\partial^2 u}{\partial z^2}, \tag{1}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - g\frac{\rho'}{\rho_0} + \nu_\mathrm{h}\frac{\partial^2 w}{\partial x^2} + \nu_\mathrm{v}\frac{\partial^2 w}{\partial z^2}, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{3}$$