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Extension of VIKOR method for decision making problem with interval numbers

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ABSTRACT

The VIKOR method was developed for multi-criteria optimization of complex systems. It determines the compromise ranking list and the compromise solution obtained with the initial (given) weights. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multi-criteria ranking index based on the particular measure of "closeness" to the "ideal" solution. The aim of this paper is to extend the VIKOR method for decision making problems with interval number. The extended VIKOR method's ranking is obtained through comparison of interval numbers and for doing the comparisons between intervals, we introduce α as optimism level of decision maker. Finally, a numerical example illustrates and clarifies the main results developed in this paper.

1. Introduction

Multi-criteria optimization is the process of determining the best feasible solution according to the established criteria (representing different effects). Practical problems are often characterized by several non-commensurable and conflicting criteria and there may be no solution satisfying all criteria simultaneously. Thus, the solution is a set of non-inferior solutions, or a compromise solution according to the decision maker's preferences. The compromise solution was established by Yu [1] and Zeleny [2] for a problem with conflicting criteria and it can be helping the decision makers to reach a final solution. The compromise solution is a feasible solution, which is the closest to the ideal, and compromise means an agreement established by mutual concessions.

A multi attribute decision making (MADM) problem can be defined as:

	C_1	C_2	$\dots$	C_n
A_1	f_{11}	f_{12}	$\dots$	f_{1n}
A_2	f_{21}	f_{22}	$\dots$	f_{2n}
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
A_m	f_{m1}	f_{m2}	$\dots$	f_{mn}

$$W = [w_1, w_2, \dots, w_n]$$

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where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance is measured, f_{ij} is the rating of alternative A_i with respect to criterion C_j, w_j is the weight of criterion C_j [3–5].

In classical MCDM methods, the ratings and the weights of the criteria are known precisely, whereas in the real world, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of a decision maker or expert are so precise. For example, human judgment including preferences is often vague and decision maker (DM) cannot estimate his preference with exact numerical values. In these situations, determining the exact value of the attributes is difficult or impossible. So, to describe and treat imprecise and uncertain elements present in a decision problem, fuzzy and stochastic approaches are frequently used. In the literature, in the works of fuzzy decision making [6–8], fuzzy parameters are assumed to be with known membership functions and in stochastic decision making [9–12] parameters are assumed to have known probability distributions. However, in reality to a decision maker (DM) it is not always easy to specify the membership function or probability distribution in an inexact environment. At least in some of the cases, the use of interval numbers may serve the purpose better. An interval number can be thought as an extension of the concept of a real number and also as a subset of the real line $\Re$ [13]. However, in decision problems its use is not much attended as it merits.

Recently, Jahanshahloo et al. [14] have extended TOPSIS method to solve decision making problems with interval data. According to a comparative analysis of VIKOR and TOPSIS written by Opricovic and Tzeng [15], VIKOR method and TOPSIS method use different aggregation functions and different normalization methods. TOPSIS method is based on the principle that the optimal point should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). Therefore, this method is suitable for cautious (risk avoider) decision maker(s), because the decision maker(s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible. Besides, computing the optimal point in the VIKOR is based on the particular measure of "closeness" to the PIS. Therefore, it is suitable for those situations in which the decision maker wants to have maximum profit and the risk of the decisions is less important for him. Therefore, we extend the concept of VIKOR method to develop a methodology for solving MADM problems with interval numbers.

The VIKOR method is presented in the next section. In section 3, extended VIKOR method is introduced and a new method is proposed for interval ranking on the basis of decision maker's optimistic level. In Section 4, an illustrative example is presented to show an application of extended VIKOR method. Finally, conclusion is presented.

2. VIKOR method

The VIKOR method was introduced as one applicable technique to be implemented within MCDM problem and it was developed as a multi attribute decision making method to solve a discrete decision making problem with non-commensurable (different units) and conflicting criteria [15,16]. This method focuses on ranking and selecting from a set of alternatives, and determines compromise solution for a problem with conflicting criteria, which can help the decision makers to reach a final solution. The multi-criteria measure for compromise ranking is developed from the L_P*-metric* used as an aggregating function in a compromise programming method [1,2].

Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The various m alternatives are denoted as $A_1, A_2, \ldots, A_m$. For alternative A_i, the rating of the jth aspect is denoted by f_{ij}, i.e. f_{ij} is the value of jth criterion function for the alternative A_i; n is the number of criteria. Development of the VIKOR method is started with the following form of L_P*-metric*:

$$L_{pi} = \left\{ \sum_{j=1}^{n} [(f_j^* - f_{ij})/(f_j^* - f_j^-)]^p \right\}^{1/p} \quad 1 \leqslant p \leqslant \infty; \quad i = 1, 2, \ldots, m. \tag{1}$$

In the VIKOR method $L_{1,i}$ (as S_i) and $L_{\infty,i}$ (as R_i) are used to formulate ranking measure. The solution obtained by min S_i is with a maximum group utility ("majority" rule), and the solution obtained by min R_i is with a minimum individual regret of the "opponent".

The compromise ranking algorithm of the VIKOR method has the following steps:

(a) Determine the best f_j^* and the worst f_j^- values of all criterion functions $j = 1,2,\ldots,n$. If the jth function represents a benefit then:

$$f_j^* = \max_i f_{ij}, \quad f_j^- = \min_i f_{ij} \tag{2}$$

(b) Compute the values S_i and R_i; $i = 1,2,\ldots,m$, by these relations:

$$S_i = \sum_{j=1}^{n} w_j (f_j^* - f_{ij})/(f_j^* - f_j^-), \tag{3}$$

$$R_i = \max_j w_j (f_j^* - f_{ij})/(f_j^* - f_j^-), \tag{4}$$

where w_j are the weights of criteria, expressing their relative importance.