



Asymptotic stability of positive equilibrium solution for a delayed prey–predator diffusion system

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ARTICLE INFO

Article history:

Received 8 October 2007

Received in revised form 27 March 2009

Accepted 30 March 2009

Available online 16 April 2009

Keywords:

Prey–predator system

Time delay

Diffusion effects

Dirichlet boundary conditions

Stability

Hopf bifurcation

ABSTRACT

This article is concerned with a delayed Lotka–Volterra two-species prey–predator diffusion system with a single discrete delay and homogeneous Dirichlet boundary conditions. By applying the implicit function theorem, the asymptotic expressions of positive equilibrium solutions are obtained. And then, the asymptotic stability of positive equilibrium solutions is investigated by linearizing the system at the positive equilibrium solutions and analyzing the associated eigenvalue problem. It is demonstrated that the positive equilibrium solutions are asymptotically stable when the delay is less than a certain critical value and unstable when the delay is greater than this critical value. In addition, it is also found that the system under consideration can undergo a Hopf bifurcation when the delay crosses through a sequence of critical values. Finally, to verify our theoretical predictions, some numerical simulations are also included.

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1. Introduction

In recent years, population models appearing in various fields of mathematical biology have been proposed and studied extensively. For example, when there are only a prey species and a predator species in a certain ecological community, under the assumptions that the growth of prey species will be governed by the logistic equation in the absence of predator species and the predator will tend to extinction when there are no prey species, Lotka [1] and Volterra [2] proposed the the following well-known Lotka–Volterra two-species prey–predator system

$$\begin{cases} \frac{du}{dt} = u(t) \left[r - \frac{u(t)}{K} \right] - bu(t)v(t), \\ \frac{dv}{dt} = v(t) [-D + cu(t)], \end{cases} \quad (1.1)$$

where $u(t)$ and $v(t)$ represent the densities of prey and predator at time t , respectively; the positive constant r and K denote the intrinsic growth rate and carrying capacity of prey species, respectively; $b > 0$ is the predation coefficient of predator; $D > 0$ denotes the death rate of predator and $c > 0$ is the transformation rate of prey to predator.

In fact, there are probably many prey species in an ecological community and the predator species can still survive in the absence of some of prey species. Therefore, if we consider only one of prey species and assume the growth of predator is still

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¹ Supported by “QingLan” Talent Engineering of Lanzhou Jiaotong University (QL-05-20A).

governed by the logistic equation in the absence of this prey species, then in this case the two-species prey–predator system can be described by the following Lotka–Volterra system of ordinary differential equations (ODEs)

$$\begin{cases} \frac{du}{dt} = u(t)[r_1 - a_{11}u(t) - a_{12}v(t)], \\ \frac{dv}{dt} = v(t)[r_2 + a_{21}u(t) - a_{22}v(t)], \end{cases} \tag{1.2}$$

where the positive constants r_1 and r_2 denote the intrinsic growth rates of prey and predator, respectively, and $a_{ij}(i, j = 1, 2)$ are all positive constants.

In proposing system (1.2), it is assumed that prey and predator species are spatially homogeneous and the future states of system are determined only by the current ones. In general, the species is spatially heterohomogeneous and hence individuals will tend to migrate towards regions of lower population density to add the possibility of survival [3]. Meanwhile, most of species need take proper time to become maturity and also need appropriate gestation period to reproduce [4–6,3]. Thus, a more realistic model reflecting the growth dynamics of prey–predator two species should be described by the following delayed Lotka–Volterra prey–predator diffusion system

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} = d_1 \Delta u(t,x) + u(t,x)[r_1 - a_{11}u(t - \tau_{11},x) - a_{12}v(t - \tau_{12},x)], \\ \frac{\partial v(t,x)}{\partial t} = d_2 \Delta v(t,x) + v(t,x)[r_2 + a_{21}u(t - \tau_{21},x) - a_{22}v(t - \tau_{22},x)], \end{cases} \quad t > 0, x \in \Omega, \tag{1.3}$$

where Δ is Laplace operator and $\Omega \subset \mathbb{R}^n$ is a bounded open domain with smooth boundary $\partial\Omega$; positive constants d_1 and d_2 denote the diffusion coefficients of prey and predator, respectively; τ_{11} and τ_{22} represent the gestation period of prey and predator species, respectively; τ_{12} is the growth time for a predator larva to possess the predation ability and τ_{21} is the growth time for a prey larva to become esculent.

System (1.3) with Neumann boundary conditions has been investigated extensively by many researchers (see, for example, [7–9] and the references cited therein). For example, Kuang and Smith [7,8] investigated the global stability of the delayed Lotka–Volterra system with Neumann boundary conditions. Recently, under the assumptions that $\tau_{ij} = \tau(i, j = 1, 2)$, by regarding the delay τ as the bifurcation parameter, Yan [9] investigated the stability of positive constant equilibrium solutions and Hopf bifurcations of nonconstant periodic solutions arisen from the increase of delay for system (1.3) with homogeneous Neumann boundary conditions. It is shown that the positive constant equilibrium solutions are asymptotically stable when τ is less than a certain critical value and unstable when τ is greater than this critical value, and system (1.3) can undergo a Hopf bifurcation when τ crosses through a sequence of critical values. In addition, by using the normal form theory and center manifold reduction for partial functional differential equations (PFDEs), the direction of Hopf bifurcation and the stability of bifurcated periodic solutions occurring through Hopf bifurcations are also investigated.

When the environment is surrounded by a totally unfavorable region in which the population density cannot attain positive values, the homogeneous Dirichlet boundary and initial value conditions

$$\begin{cases} u(t,x) = v(t,x) = 0, \quad x \in \partial\Omega, \quad t \geq 0, \\ u(t,x) = \phi(t,x), \quad v(t,x) = \psi(t,x), \quad (t,x) \in [-\tau, 0] \times \bar{\Omega} \end{cases} \tag{1.4}$$

should be imposed on system (1.3), where

$$\phi, \psi \in C = C[-\tau, 0], \quad H_0^1(\bar{\Omega}) \quad \text{and} \quad \tau = \max_{1 \leq i, j \leq 2} \{\tau_{ij}\}.$$

As pointed out by Huang [10], it is more difficult to handle the Dirichlet boundary conditions because in this case the nontrivial equilibrium solutions or nontrivial periodic solutions (if they exist) are spatially heterohomogeneous. Meanwhile, it is also very complicated and almost impossible to analyze completely the effects of delays on the dynamical behaviors of systems with multiple different delays. In view of these difficulty, we firstly observe the effects of delays on the dynamical behaviors of system (1.3) and (1.4) by the numerical methods for partial differential equations. To this end, let $\Omega = (0, \pi)$ in system (1.3) and (1.4), and consider the following system:

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} = 0.1 \frac{\partial^2 u(t,x)}{\partial x^2} + u(t,x)[0.2 - 0.1u(t - \tau_{11},x) - 0.2v(t - \tau_{12},x)], \\ \frac{\partial v(t,x)}{\partial t} = 0.2 \frac{\partial^2 v(t,x)}{\partial x^2} + v(t,x)[0.3 + 0.2u(t - \tau_{21},x) - 0.3v(t - \tau_{22},x)], \\ t > 0, \quad 0 < x < \pi, \\ u(t,x) = v(t,x) = 0, \quad x = 0, \pi, \quad t \geq 0, \\ u(t,x) = v(t,x) = 0.1(1 + \frac{t}{\tau}) \sin x, \quad (t,x) \in [-\tau, 0] \times [0, \pi]. \end{cases} \tag{1.5}$$

The numerical simulations show that system (1.5) without delays has a stable positive equilibrium solution (see Fig. 1); when $\tau_{11} = 0.21, \tau_{12} = 0.15, \tau_{13} = 0.17$ and $\tau_{22} = 0.32$, the positive equilibrium solution is still stable (see Fig. 2); when $\tau_{11} = 8, \tau_{12} = 5, \tau_{13} = 7$ and $\tau_{22} = 13$, the positive equilibrium solution is unstable and there is an oscillation near the positive equilibrium solution (see Fig. 3).

As the special case of system (1.3) and (1.4), we see that sufficiently small delays in system (1.5) preserve the stability of the positive equilibrium solutions and large delays destabilize the positive equilibrium solution, cause oscillation. It is

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