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Unscented Kalman filter for time varying spectral analysis of earthquake ground motions

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Abstract

A novel parametric time-domain method for time varying spectral analysis of earthquake ground motions is presented. Based upon time varying autoregressive moving average (ARMA) modeling of earthquake ground motion, unscented Kalman filter (UKF) is used to estimate the time varying ARMA coefficients. Then, time varying spectrum is yielded according to the time varying ARMA coefficients. Analysis of the ground motion record El Centro (1940, N–S) shows that compared to Kalman filter (KF) based method, short-time Fourier transform (STFT) and wavelet transform (WT), UKF based method can more reasonably represent the distribution of the seismic energy in time–frequency plane, which ensures its better ability to track the local properties of earthquake ground motions and to identify the systems with nonlinearity. Analysis of the seismic response of a building during the 1994 Northridge earthquake shows that UKF based method can be potentially a useful tool for structural damage detection and health monitoring. Lastly, it is found that the theoretical frequency resolving power of ARMA models usually neglected in some studies has considerable effect on time varying spectrum and it is one of the key factors for ARMA modeling of earthquake ground motion.

Keywords: Earthquake ground motions; Time varying spectrum; Autoregressive moving average model; Unscented Kalman filter; Kalman filter

1. Introduction

Recent research studies have indicated that the conventional properties of earthquake ground motions (e.g., the properties of amplitude, frequency and duration), are not so enough as to thoroughly influence the characteristics of structural seismic response and other properties (e.g., the nonstationary properties in amplitude and frequency contents) can also influence the structural seismic response significantly [1]. To describe these nonstationary properties efficiently, some notions have been introduced for practical purpose (e.g., the zero-crossing rate is used to describe the nonstationarity in frequency contents and the time varying spectrum is

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used to describe the nonstationarity both in amplitude and frequency contents). Among these notions, the time varying spectrum is thought as the most suitable one, for it can provide a proper description of energy distribution in time and frequency domains. The modeling and estimation of time varying spectrum has been an open issue for the characterization of earthquake ground motions.

In most cases, the methods for estimating time varying spectrum can be classified as nonparametric and parametric. The nonparametric methods chiefly comprise time–frequency analysis (e.g., short-time Fourier transform, spectrogram and Wigner–Ville distribution [2,3]), higher order spectral analysis, evolutionary spectrum, wavelet transform and Hilbert–Huang transform (HHT), etc., [4–8]. Though successful applications of them have been reported, additional modeling processes are needed if they are used to study the attenuation laws of time varying spectrum and to synthesize ground motions for seismic design considering the nonstationary properties. The parametric methods mainly refer to the modern spectral estimation methods based on autoregressive moving average (ARMA) models. They complement the nonparametric counterparts and offer advantages such as representation parsimony, improved accuracy, resolution, and tracking, thus they are more suitable for modeling and attenuation laws determination of time varying spectrum and selection or synthesis of ground motions for seismic design [9–11].

The parametric methods consists of four steps usually, i.e., (1) time varying ARMA modeling of earthquake ground motions, (2) state-space representations of time varying ARMA models, (3) recursive estimation of time varying coefficients (states), and (4) time varying spectrum estimation according to the time varying coefficients. The time varying coefficients estimation is the core of parametric methods and it has significant influence on precision and reliability of estimation results.

The commonly used methods for time varying coefficients estimation are Kalman filter family methods [12–16], grid based filter and particle (Monte Carlo) filter [15,17]. The Kalman filter family methods include standard Kalman filter (KF) [11–16], square root Kalman filter (SRKF) [16], extended Kalman filter (EKF) [14–16], least mean square (LMS) method, recursive least square (RLS, or so called forgetting factor) method [14,15] and Kalman (Rauch–Tung–Striebel) smoother [15,16]. They are optimal methods in the sense of least square error only when the driving noises are Gaussian and the state-space models are linear. While in cases that the noises are non-Gaussian or the models are nonlinear, the Gaussian assumption of the non-Gaussian noises and the linearization of the nonlinear models can lead to suboptimal performance and sometimes divergence of the filter. On the contrary, the grid based filter and particle filter are universal methods in the framework of Bayesian principle and they can handle non-Gaussian noises and nonlinear models cases. These two methods use a large number of random sample points drawn from a prior proposal distribution to recursively estimate the states and the corresponding covariance, thus the computational complexity of them increases drastically as the dimensionality of states increases and the estimation precision relies on the knowledge of the prior distribution of states, which has blocks their utilization in practice.

To address above problems, considerable attention has been paid to the improved Kalman filter, the unscented Kalman filter (UKF) which uses a minimal set of determinate sample points (Sigma points) to completely capture the true mean and covariance of the states via unscented transformation and has been proved to be a universal, simple and precise method [15,18–21]. Thus in this paper the UKF based method is employed to estimate the time varying coefficients of ARMA models.

2. ARMA modeling and time varying spectrum estimation

2.1. Time varying ARMA modeling of earthquake ground motion

Generally the earthquake ground motion y_k as a realization of a nonstationary random process can be formulated as following time varying ARMA (p, q) model [9–11,22–24]:

$$y_k - \phi_{1,k}y_{k-1} - \cdots - \phi_{p,k}y_{k-p} = e_k - \theta_{1,k}e_{k-1} - \cdots - \theta_{q,k}e_{k-q}, \tag{1}$$

where e_k is Gaussian noise and $e_k \sim N(0,\sigma_k^2)$. σ_k is also called envelope function. The variables $\{\phi_{i,k}i = 1,2,\ldots,p\}$ and $\{\theta_{i,k}\ i = 1,2,\ldots,q\}$ are the time varying autoregressive (AR) coefficients and moving average (MA) coefficients, respectively. The subscript k denotes the instant $t = k\Delta t$ where Δt is the sampling time.