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Stochastic heat conduction analysis of a functionally graded annular disc with spatially random heat transfer coefficients

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Abstract

The mean and variance of the temperature are analytically obtained in a functionally graded annular disc with spatially random heat transfer coefficients (HTCs) on the upper and lower surfaces. This annular disc has arbitrary variations in the HTCs (i.e., arbitrary thermal interaction with the surroundings) and gradient material composition only along the radial direction and is subjected to deterministic axisymmetrical heating at the lateral surfaces. The stochastic temperature field is analysed by considering the annular disc to be multilayered with spatially constant material properties and spatially constant but random HTCs in each layer. A type of integral transform method and a perturbation method are employed in order to obtain the analytical solutions for the statistics. The correlation coefficients of the random HTCs are expressed in the form of a linear function with respect to the radial distance as a non-homogeneous random field of discrete space. Numerical calculations are performed for functionally graded annular discs composed of stainless steel and ceramic, which comprise two types of material composition distributions. The effects of the magnitude of the means of HTCs, volume fraction distributions of the constitutive materials and correlation strengths of the HTCs on the standard deviation of the temperature are discussed.

Keywords: Perturbation method; Random field; Functionally graded material; Heat conduction; Random heat transfer coefficient; Annular disc

1. Introduction

The heat conduction problems of discs heated by their surroundings are important due to their wide range of industrial applications, such as annular fins, turbine discs and brake disc rotors. Therefore, a number of theoretical studies on them pertaining to the investigation of their associated thermal stresses have thus far been reported. These include non-axisymmetrically heated discs [1,2], variable thickness discs [1,3,4] and non-homogeneous discs such as functionally graded materials (FGMs) [1,2,5,6]. However, to simplify the

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analyses, almost all the studies were conducted with the following assumption: the upper and lower surfaces of the discs are insulated or heat is dissipated with uniform heat transfer coefficients (HTCs) throughout the surfaces.

In actual thermal environments, the HTCs of object surfaces are known to vary spatially and depend heavily on the motion of the surrounding media and the surface properties including surface asperities [7,8]. Obviously, the effect of the spatial change in the HTCs on the temperature fields is large in objects with a wide heat transfer area per unit volume, such as thin strips, plates and discs. In addition, it is well known that the HTC is the most uncertain factor in an analysis that considers the heat dissipation to the surroundings [9]. This is because its value depends on many factors, as mentioned above, and many diverse empirical correlations exist for it. Therefore, it is logical to statistically deal with the heat conduction problems for such thin objects using stochastic models.

Generally, the spatial change in HTCs significantly affects the temperature distribution in objects, thereby causing a considerable change in the shape of the thermal stress distribution. Sugano [10] and Chen [11] investigated the transient temperature and thermal stresses in a rectangular plate with a unidirectional variation in the HTCs on the upper and lower surfaces and emphasized the importance of the location dependence of the HTCs in thermal stress analyses. Subsequently, Lee et al. [12] investigated the transient coupled thermoelasticity of an annular fin with variable HTCs along the radial direction, and Sugano et al. [13] performed the material design of functionally graded rotating discs considering the spatial change in HTCs on the surfaces. Moreover, Heggs et al. [14] and Barrow [15] analysed the temperature fields in an annular fin and a rectangular fin with variable surface HTCs and discussed the effect of variable HTCs on their fin efficiencies. However, the uncertainties in the HTCs were not considered in their studies.

There are only a few stochastic studies on the heat conduction problem that considers the spatial or temporal randomness of HTCs. Madera [16] and Emery [17] analysed the stochastic heat conduction problem for a rectangular fin in which the HTC on the extended surface is expressed as a stochastic process and a random field, respectively. Kuznetsov [9] analysed the stochastic heat conduction problem for an infinite strip with random HTC whose normalized correlation function is equal to 1 everywhere. However, these studies assumed a homogeneous material. To the best of the author's knowledge, no study on the heat conduction in a non-homogeneous body with random HTCs has thus far been reported. If the statistics of temperature in a non-homogeneous body with an arbitrary non-homogeneity are obtained in explicit forms, they are quite useful because they can be applied in the estimation of the reliability of FGMs with thermal stress relaxation, which have been developed recently.

The main objective of this study is to analytically derive the second-order statistics, i.e., the mean and variance of the temperature in an axisymmetrically heated functionally graded annular disc with spatially random HTCs on the upper and lower surfaces. Here, the functionally graded disc under consideration is the one in which the material composition is a function of the radial coordinate only. The HTCs are assumed to vary arbitrarily along the radial direction and disperse randomly with some random amplitude around their means as a non-homogeneous random field. Vodicka's method [18], which is a type of integral transform method, and a perturbation method are employed to obtain the analytical solutions for the statistics.

Numerical calculations are performed for functionally graded annular discs having increasing HTC means along the radial direction, which comprise two types of material composition distributions. We discuss the effects of the magnitude of the means of the HTCs, volume fraction distributions of the constitutive materials and correlation strengths of the HTCs on the standard deviation (SD) of temperature. Additionally, the derived analytical solution is verified and its applicability is examined by comparing with the results obtained by direct Monte Carlo simulation.

2. Analytical model

Let us consider a functionally graded annular disc with an inner radius a, outer radius b and constant thickness d in which the HTCs on the upper and lower surfaces, η and χ, respectively, vary arbitrarily only along the radial (r-axis) direction. The upper and lower surfaces of this disc make contact with the surrounding media with deterministic temperatures T_u and T_l, respectively. It is assumed that the disc material is isotropic but non-homogeneous and the material properties – density ρ, specific heat c and thermal conductivity λ – are