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A smoothing Levenberg–Marquardt method for the extended linear complementarity problem ☆

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ABSTRACT

We consider the extended linear complementarity problem (XLCP), of which the linear and horizontal linear complementarity problems are two special cases. We reformulate the XLCP to a smooth equation by using some smoothing functions and propose a Levenberg–Marquardt method to solve the system of smooth equation. The global convergence and local superlinear convergence rate are established under certain conditions. Numerical tests show the effectiveness of the proposed algorithm.

1. Introduction

The extended linear complementarity problem (XLCP) introduced by Mangasarian and Pang [1], is to find a pair of vectors x and y in R^n such that

$$Mx - Ny \in \mathscr{P}, \quad x \geqslant 0, \quad y \geqslant 0, \quad \langle x, y \rangle = 0, \tag{1}$$

where M and N are two real matrices of order $m \times n$, $\mathscr{P}$ is a polyhedral set in R^m and $\langle \cdot, \cdot \rangle$ denotes the usual inner product. Throughout this paper, we assume that the feasible set of XLCP is nonempty

$$\{(x, y) | Mx - Ny \in \mathscr{P}, x \geqslant 0, y \geqslant 0\} \neq \emptyset.$$

This problem arises in engineering, equilibrium modelling and optimization problems, and is a unifying framework of various linear complementarity problems, such as the linear complementarity problems (LCP), see [30] the horizontal linear complementarity problem (HLCP) and the mixed linear complementarity problem (MLCP), see [2].

Over the past decades, many equivalent reformulation forms of the XLCP have been proposed by some researchers. For example, Mangasarian and Pang [1] and Gowda [3] considered the equivalent relationship between XLCP and the following bilinear program (BLP):

$$\min \langle x, y \rangle \quad \text{s.t. } Mx - Ny \in \mathscr{P}, \quad x \geqslant 0, \quad y \geqslant 0. \tag{2}$$

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Andreani and Martinez [4] and Solodov [5] considered reformulating the XLCP as an unconstrained or a nonnegative constrained optimization problem.

The motivation for our study comes from recent smoothing methods for complementarity, variational inequality and mathematical programming problems, see, for example [6–11,25]. Especially, the smoothing methods have been proved to be succeed in solving complementarity problems include the HLCP [12] and VLCP [13] in recent years. However, as we observe, there is few smoothing method available for the XLCP given by (1).

The aim of this paper is to propose a smoothing Levenberg–Marquardt method for XLCP. By using the minimum function and the Fischer–Burmeister function, we first reformulate the XLCP as a system of nonsmooth equations, and using the smoothing technique we construct the smooth operator. A smoothing Levenberg–Marquardt method is proposed to solve the system of smooth equations. Under certain conditions, we obtain the global and local convergence properties of the proposed algorithm.

This paper is organized as follows: in Section 2, we give the equivalent reformulation of the XLCP. The algorithm and global convergence is given in Section 3. The local superlinear convergence is proved in Section 4. In Section 5, we report our numerical tests to show the effectiveness of our algorithm. The conclusion is presented in Section 6.

Throughout this paper, we assume the polyhedral set $\mathscr{P}$ in R^m appearing in the statement of XLCP (1) is presented as

$$\mathscr{P} = \{u \in R^m | Au \geqslant h\},$$

where A is some $k \times m$ real matrix and $h \in R^k$.

2. Equivalent smoothing reformulation of the XLCP

In this section, we give the equivalent smoothing reformulation of the XLCP and discuss some associated properties of the reformulation. Firstly, we introduce the NCP function and the smoothing function. A function $\varphi : R^2 \to R$ is called a NCP function if it possesses the following property:

$$\varphi(a,b) = 0 \Longleftrightarrow a \geqslant 0, \quad b \geqslant 0, \quad ab = 0.$$

Two well-known NCP functions are the minimum function and the Fischer–Burmeister function [14], which are defined as follows:

$$\varphi_{FB}(a,b) = a + b - \sqrt{a^2 + b^2},$$
$$\varphi_{min}(a,b) = a + b - |a - b|.$$

Accordingly, the smoothing functions associated with φ_{FB} and φ_{min} are [6,15]:

$$\varphi_{FB}(a,b,\tau) = a + b - \sqrt{a^2 + b^2 + 2\tau^2},$$
$$\varphi_{min}(a,b,\tau) = a + b - \sqrt{(a-b)^2 + 4\tau^2}.$$

Now we will consider the equivalent reformulation of XLCP given by (1). Based on the discussion in [5], we reformulate (1) into the following equivalent system of nonlinear equation:

$$\Phi(x,y,z) := \begin{pmatrix} \Phi_{FB}(x,y) \\ AMx - ANy - b - z \\ \Phi_{min}(z,0) \end{pmatrix} = 0, \tag{3}$$

where $\Phi_{FB}(x,y) = (\varphi_{FB}(x_1,y_1), \varphi_{FB}(x_2,y_2), \cdots, \varphi_{FB}(x_n,y_n))^T \in R^n$ and $\Phi_{min}(z,0) = (\varphi_{min}(z_1,0), \varphi_{min}(z_2,0), \ldots, \varphi_{min}(z_m,0))^T \in R^n$.

By using the smoothing function, we define the smooth approximation of Φ as $\Phi_\tau : R^n \times R^n \times R^m \to R^n \times R^m \times R^m$

$$\Phi_\tau(x,y,z) := \begin{pmatrix} \Phi_{FB}(x,y,\tau) \\ AMx - ANy - b - z \\ \Phi_{min}(z,0,\tau) \end{pmatrix} \tag{4}$$

For convenience, we rewrite $w = (x^T, y^T, z^T)^T$, and accordingly we denote $\Phi(w) = \Phi(x,y,z)$, $\Phi_\tau(w) = \Phi_\tau(x,y,z)$, furthermore, we denote the corresponding merit function by

$$\Psi(w) = \frac{1}{2}\|\Phi(w)\|^2$$

and

$$\Psi_\tau(w) = \frac{1}{2}\|\Phi_\tau(w)\|^2.$$

Then the XLCP is equivalent to the following minimization problem:

$$\min \Psi(w) \tag{5}$$

with object function value zero.