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Modelling of three-dimensional mechanical systems using point coordinates with a recursive approach

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Abstract

In this paper, the dynamic simulation of constrained mechanical systems that are interconnected of rigid bodies is studied using projection recursive algorithm. The method uses the concepts of linear and angular momentums to generate the rigid body equations of motion in terms of the Cartesian coordinates of a dynamically equivalent constrained system of particles, without introducing any rotational coordinates and the corresponding rotational transformation matrix. Closed-chain system is transformed to open-chain by cutting suitable kinematical joints and introducing cut-joint constraints. For the resulting open-chain system, the equations of motion are generated recursively along the serial chains. An example is chosen to demonstrate the generality and simplicity of the developed formulation.

Keywords: Dynamic simulation; Equations of motion; Recursive projection; Mechanical systems; Mechanisms

1. Introduction

There are different formulations for the dynamic analysis of spatial mechanisms which vary in the system of coordinates used and in the way they introduce kinematical constraint equations. Each formulation has its own advantages and disadvantages depending on the application and the needs. Some formulations are developed using a two-step transformation which leads to a simple and reduced system of equations. One method [1,2] uses initially the absolute coordinate formulation where the location of each rigid body in the system is described in terms of a set of translational and rotational coordinates. Then, the equations of motion are transformed to a reduced set in terms of the relative joint variables. Another method uses initially the point coordinate formulation in which a dynamically equivalent constrained system of particles replaces the rigid bodies [3–6]. The global motion of the constrained system of particles together with the constraints imposed upon them represent both the translational and rotational motions of the rigid body. The external forces and couples acting on the body are distributed over the system of particles. Then, the equations of motion which

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are expressed in terms of the Cartesian coordinates of the particles are rederived in terms of the relative joint variables. The main disadvantage of this two-step transformation is the necessity to transform at every time step from the joint variables to the original system which is time consuming.

A recursive dynamical formulation for the dynamic analysis of planar mechanisms is presented by Attia [7]. The concepts of linear and angular momentum are used to write the rigid body dynamical equations without the need to distribute the external forces and couples over the particles. The method can be applied to recursively generate the equations of motion for open and/or closed-chain systems.

In this paper, the dynamic simulation of constrained mechanical systems that are interconnected of rigid bodies is studied using projection recursive algorithm. The method is based upon the idea of replacing the rigid body by its dynamically equivalent constrained system of particles discussed in [3–7] with essential modifications and improvements. The concepts of the linear and angular momentums are used to formulate the rigid body dynamical equations. However, they are expressed in terms of the rectangular Cartesian coordinates of the equivalent constrained system of particles. This groups the advantages of the automatic elimination of the unknown internal forces as in Newton–Euler formulation and results in a reduced system of differential-algebraic equations. Some useful geometrical relationships are used to obtain a reduced dynamically equivalent constrained system of particles.

For the closed-chain system, the system is transformed to open-chain system by cutting suitable kinematical joints and introducing the cut-joint kinematical constraints. For the resulting open-chain system, the equations of motion are generated recursively along the serial chains instead of the matrix formulation derived in [3–6]. Most of the kinematical constraints due to the kinematical joints are automatically eliminated by properly locating the equivalent particles. Examples are chosen to demonstrate the generality and simplicity of the proposed method.

2. The dynamic formulation

2.1. Construction of the equivalent system of particles

A rigid body and its dynamically equivalent constrained system of particles should have the same mass, the position of the centre of mass and the inertia tensor with respect to a body attached coordinate frame which results in 10 conditions in the spatial case. The choice of four particles (not all are laying in the same plane) results in 16 unknowns (4 masses + 12 coordinates) that should satisfy the 10 conditions. However, this choice will lead to the solution of nonlinear algebraic equations due to the quadratic form of the second moments and also it does not give the freedom to choose all the particles in important places in the mechanisms. A system of 10 particles will lead to the solution of 10 linear algebraic equations in 10 unknown masses. Also it gives the freedom of positioning the particles on the bodies in accordance with the joints that connect the bodies in order to reduce the number of particles and consequently eliminates some geometric and kinematical constraints. Therefore, a system of 10 particles is chosen to replace the rigid body as shown in Fig. 1. It constitutes

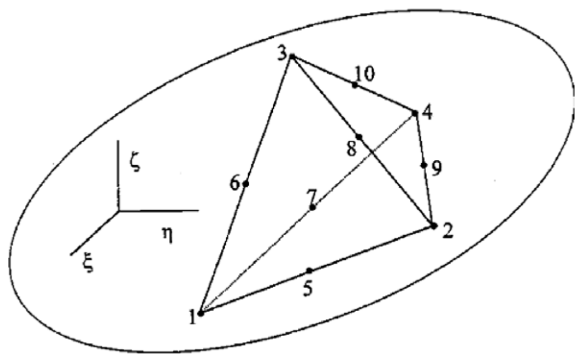

Fig. 1. The rigid body system with the equivalent system of 10 particles.