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Numerical simulation of laminar flow past a circular cylinder

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Abstract

The present paper focuses on the analysis of two- and three-dimensional flow past a circular cylinder in different laminar flow regimes. In this simulation, an implicit pressure-based finite volume method is used for time-accurate computation of incompressible flow using second order accurate convective flux discretisation schemes. The computation results are validated against measurement data for mean surface pressure, skin friction coefficients, the size and strength of the recirculating wake for the steady flow regime and also for the Strouhal frequency of vortex shedding and the mean and RMS amplitude of the fluctuating aerodynamic coefficients for the unsteady periodic flow regime. The complex three dimensional flow structure of the cylinder wake is also reasonably captured by the present prediction procedure.

Keywords: Laminar flow; Circular cylinder; Vortex shedding; Unsteady RANS procedure; Implicit finite volume method; Three-dimensional flow

1. Introduction

In spite of extensive experimental and numerical studies almost over a century, flow around a circular cylinder still remains a challenging problem in fluid mechanics, where intensive investigations are continued even today to understand the complex unsteady dynamics of the cylinder wake flow. Cross-flow normal to the axis of a stationary circular cylinder and the associated problems of heat and mass transport are encountered in a wide variety of engineering applications. Both experimental measurements and numerical computations have confirmed the onset of instability of the wake flow behind a cylinder beyond a critical Reynolds number, leading finally to a kind of periodic flow identified by definite frequencies, well-known in the literature as the Von Karman vortex street. In case of laminar flow past cylinders with regular polygonal cross-section, the flow usually separates at one or more sharp corners of the cross-section geometry itself, forming a system of vortices in the wake on either side of the mid symmetry plane. On the other hand, for a circular cylinder, where the point of flow separation is decided by the nature of the upstream boundary layer, the physics of the flow is much more complex than what its relatively simple shape might suggest.

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As the flow Reynolds number ($Re = UD/v$) based on the free stream velocity (U), cylinder diameter (D) and kinematic viscosity v of the fluid, changes from a creeping laminar flow value of the order of 0.1 to a turbulent flow of the Re of a million or even higher, variety of physical complexities start taking place. Steady laminar flow exists at Reynolds number between 5 and 40 with a pair of symmetric counter-rotating vortices formed behind the cylinder. Beyond a critical value of Re, depending on the other flow disturbances, a transverse oscillation sets in with loss of flow symmetry and vortices are shed from the cylinder surface. Between $Re = 190$ and 260, wakes of two dimensional cylinders are observed to become susceptible to a primary instability mechanism which leads to the amplification of three dimensional disturbances and eventually to the development of strong streamwise oriented vortical structures. These three-dimensional disturbances alter the structure and evolution of the wake vortices and as a result, even at low Reynolds number, two-dimensional simulations often fail to accurately predict even gross flow quantities like mean drag coefficient and root mean square (RMS) of the lift fluctuation.

2. Characteristic regimes of flow past a circular cylinder

Recently Zdravkovich [1], in an excellent monograph, has compiled almost all the experimental, analytical and numerical simulation data on flow past cylinders, available since 1938 and systematically classified this challenging flow phenomenon into five different flow regimes based on the Reynolds number. In the present study, the computation is restricted only to the first few regimes designated by Zdravkovich as (1) creeping laminar state (L1) of flow ($0 < Re < 4$), (2) laminar flow (L2) with steady separation ($4 < Re < 48$) forming a symmetric contra-rotating pair of vortices in the near wake, (3) laminar flow (L3) with periodic vortex shedding ($48 < Re < 180$) and finally (4) part of the transition-in-wake (TrW) regime ($180 < Re < 400$) when the three-dimensional instabilities lead to the formation of streamwise vortex structure. Computations are carried out in the present work in the range of $0.1 < Re < 400$ for comparison with available measurement data or other computation results for both steady and unsteady flow situations. All the computations use an implicit pressure-based finite volume Navier–Stokes algorithm (RANS3D) for time-accurate prediction of the flow. This flow solution algorithm, originally proposed by the last author [2], has been further developed [3–5] at the CTFD Division, NAL, Bangalore during the last 10 years. The whole flow domain covering the L1–TrW regimes has first been computed under the assumption of two-dimensional flow. Later, in order to understand the effect of the three-dimensional disturbances on the complex wake vortex dynamics, three-dimensional analysis have been carried out at five different Reynolds number covering part of the L3 and TrW regimes, already reported in unclassified literature by a few researchers [6–9]. The present computation results have been compared to available measurement data and/or other computation results for the wake flow structures, the temporal evolution of drag coefficient, the variation of separation angle, stagnation pressure and base pressure behind the cylinder as the Reynolds number increases and also for the frequency and amplitude of the fluctuating forces arising out of the phenomenon of vortex shedding in the relevant flow regime.

3. Numerical simulation methodology

3.1. Governing equation

The Navier–Stokes equations for unsteady laminar incompressible flow may be written in a compact form in non-orthogonal curvilinear coordinates used in the present simulation as follows:

Momentum transport for the cartesian velocity component U_i:

$$\frac{\partial(\rho U_i)}{\partial t} + \frac{1}{J}\frac{\partial}{\partial x_j}\left[(\rho U_i U_k \beta_i^j) - \frac{\mu}{J}\left(\frac{\partial U_i}{\partial x_m}B_m^j + \frac{\partial U_k}{\partial x_m}\beta_i^m \beta_k^j\right) + P\beta_i^j\right] = S_{U_i}, \tag{1}$$

where J is the transformation Jacobian between cartesian and the curvilinear coordinates, β_j^i and B_j^i are the relevant metric coefficients related to the geometrical transformation, P is the pressure and j, k and m are used as repeated summing indices along the grid directions. U_i is the mean cartesian velocity solved for along the 'i-th' direction. These three momentum equations are further supplemented by the continuity equation as follows: