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Modeling of water surface profile in subterranean channel by differential quadrature method (DQM)

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Abstract

This study, investigates the hydraulic of flow in a subterranean channel headspring. The continuity and momentum equations of flow in porous media considering real conditions were used and the basic equation of flow in a subterranean channel was resulted. This equation is very similar to the spatially varied flow with increasing discharge. An equation, defining the hydraulic parameters of a subterranean channel section was adopted. Then differential quadrature method (DQM), was applied to the equation of flow in subterranean channel, consequently the water surface profile was resulted. To illustrate the rightness of model, the hydraulic parameters of flow in the Gavgard branch of the Joopar Goharriz Qanat were measured and the water surface profile was determined. This water surface profile was compared to the water surface profile computed by the model, which are in good agreement.

Keywords: Subterranean channel; Qanat; Differential quadrature method; Water surface profile; Porous media

1. Introduction

Simulation of flow in subterranean channels (Qanats) is of great importance in underground flow analysis. Flow in subterranean channels can be established, in steady or unsteady state, uniform or varied situations. It varies along the Qanat headspring and become uniform at the end of this region. In the headspring region, water is infiltrated from aquifer into the subterranean channel. The rate of inflow to the channel depends on the hydraulic gradient between water table in aquifer and water surface in subterranean channel, and hydraulic conductivity of porous media. Modeling of this process creates a complicated mathematical model and therefore complicated equation. The steady varied flow condition in subterranean channels is governed, by a classic first-order nonlinear ordinary differential equation. This nonlinear equation does not have analytical solution, but numerical techniques can be helpful to solve this equation. In seeking an efficient discretization technique to obtain accurate

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numerical solution with a considerable small number of grid points, the DQM is used to solve numerically this classic first-order nonlinear ordinary differential equation [1].

DQM was invented by Bellman in 1971 based on simple analogy with integral quadrature [2,3]. The results of Bellman's research were used in diverse areas of computational mechanics and have been claimed as a highly accurate scheme with minimum computational efforts by many researchers [4].

Application of DQM in various areas such as structural analysis, static and dynamic analysis of plates, fluid mechanics and transport phenomena led to further improvement of the method. DQM have been implemented by several researchers for solving the Navier–Stoke's equation in computational fluid dynamic (Shu et al.,1994, 1995, 1996; Jian et al., 1998; Shu, 2000; Ying et al., 2004 [3,5,6]). It seems that the capabilities of DQM for simulation of free surface flow in subterranean channel have not been explored yet. In this study, the differential quadrature method is used to solve numerically the steady varied flow equation in subterranean channel. The solution of this equation allows the tracing of longitudinal water surface profile. Accurate tracing of water surface profile along the longitudinal section of subterranean channel is required for variety of purposes including ridding flood flow on ground surface around the subterranean channel.

2. Subterranean channel

Subterranean channel is an underground mild slope channel that has been bounded with porous media. The subterranean channels include two saturated and unsaturated zones. At the channel headspring, porous media is saturated with high water table compare to the channel water level. Due to the hydraulic gradient between saturated zone and channel water level, water is penetrating continuously to the channel. The channel flow rate increases along the saturated zone of subterranean channel and it becomes maximum at the end of this zone. Longitudinal and cross section of this channel are shown in Fig. 1.

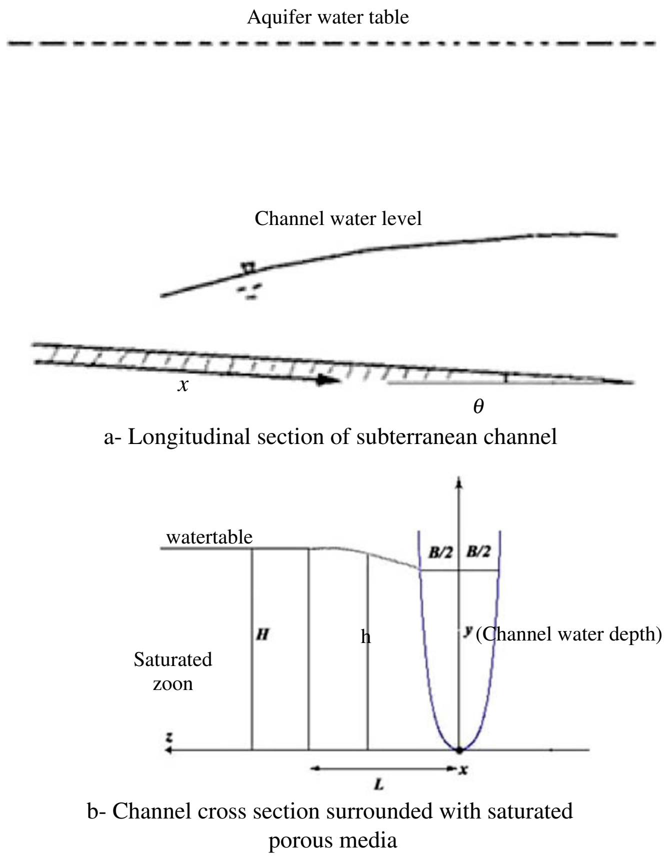

Fig. 1. (a) Longitudinal and (b) cross section of a subterranean channel in the saturated zone.