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Global exponential stability of Hopfield neural networks with distributed delays

Jie Zhou *, Shuyong Li, Zhiguo Yang

College of Mathematics and Software Science, Sichuan Normal University, Chengdu 610068, PR China

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Abstract

In this paper, dynamical behaviors of Hopfield neural networks system with distributed delays were studied. By using contraction mapping principle and differential inequality technique, a sufficient condition was obtained to ensure the existence uniqueness and global exponential stability of the equilibrium point for the model. Here we point out that our methods, which are different from previous known results, base on the contraction mapping principle and inequality technique. Two remarks were also worked out to demonstrate the advantage of our results.

Keywords: Global exponential stability; Equilibrium point; Distributed delays; Hopfield neural networks

1. Introduction

Hopfield neural networks [1] have been extensively studied in past years and found many applications in different areas such as pattern recognition, associative memory and combinatorial optimization. Such applications heavily depend on the dynamical behaviors. Thus, the analysis of the dynamical behaviors is necessary step for practical design of neural networks.

One of the most investigated problems in dynamical behaviors of Hopfield neural networks is that of the existence, uniqueness and global stability of the equilibrium point. The property of the global stability which means that the domain of attraction of the equilibrium point is the whole space, is of importance from the theoretical as well as application of view in several field. Many researchers have studied the global stability of the Hopfield neural networks with distributed delays [2–6]. They gave the sufficient conditions that guaranteed the existence, uniqueness and global asymptotic stability of the equilibrium point. But they did not give out the convergence rate. Recently, some authors studied the global exponential stability of Hopfield neural networks with distributed delays. For instance, in 2003, Zhang [7] considered the following system of integro-differential equations as a model for Hopfield neural networks with continuously distributed delays:

* Corresponding author.
E-mail address: jiezhou198208@126.com (J. Zhou).

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$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_{ij}(t-s) g_j(x_j(s)) \mathrm{d}s + J_i, & t \geqslant 0, \\ x_i(s) = \varphi_i(s), & -\infty < s \leqslant 0, \quad i = 1, \ldots, n, \end{cases} \tag{1.1}$$

the kernel function $k_{ij} : [0, \infty) \to [0, \infty)(i, j = 1, \ldots, n)$ are continuous on $[0, \infty)$ with $\int_0^\infty k_{ij}(s)\mathrm{d}s = 1$, $\int_0^\infty k_{ij}(s)\mathrm{e}^{\lambda s}\mathrm{d}s = K_{ij} < \infty$, and satisfy

$$\rho(P^{-1}KL) < 1, \text{ where } P = \mathrm{diag}(c_1, \ldots, c_n), L = \mathrm{diag}(l_1, \ldots, l_n), K = (k_{ij}|b_{ij}|)_{n\times n}. \tag{1.2}$$

They derived the criteria of global exponential stability for the equilibrium point of Eq. (1.1) by the method of variation parameter and inequality technique when hypothesis (1.2) held.

In 2006, Wang et al. [8] discussed a general Hopfield neural networks system involving distributed delays and impulses:

$$\begin{cases} \dot{x}_i(t) = -a_i x_i(t) + \sum_{j=1}^{n} p_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} q_{ij} \int_0^\infty k_{ij}(s) g_j(x_j(t-s)) \mathrm{d}s + c_i, & t \geqslant 0, \ t \neq t_k, \\ \Delta x_i(t_k) = I_k(x_i(t_k)), & i = 1, \ldots, n, \ k = 1, 2, \ldots, \\ x_i(s) = \varphi_i(s), & -\infty < s \leqslant 0, \ i = 1, \ldots, n, \end{cases} \tag{1.3}$$

in which the kernels k_{ij} are real valued nonnegative functions defined on $[0, \infty)$ with $\int_0^\infty |k_{ij}(s)|\mathrm{e}^{\lambda_0 s}\mathrm{d}s < \infty$, $\lambda_0 > 0$, and satisfy

$$\begin{aligned} &\rho(F) < 1, \text{ where } F = D^{-1}(PL^f + QL^g), \\ &D = \mathrm{diag}(a_1, \ldots, a_n), \quad P = (|p_{ij}|)_{n\times n}, \quad Q = (|\, q_{ij} \,|\, k_{ij}^{+}), \quad \int_0^\infty k_{ij}(\theta)\mathrm{d}\theta \leqslant k_{ij}^{+}. \end{aligned} \tag{1.4}$$

Sufficient conditions for the existence and global exponential stability of a unique equilibrium point were established by using the fixed point theorem and differential inequality techniques.

Motivated by the above discussions, our objective in the paper is to study further the existence, uniqueness and global exponential stability of the equilibrium point of Hopfield neural networks with distributed delays. Here we point out that our methods, which are different from previous known results, base on the contraction mapping principle and inequality technique [9].

The rest of this paper is organized as follows. In Section 2, some notations and definitions are given. In Section 3, the existence and uniqueness of the equilibrium point is obtained. We derive a sufficient condition for GES of the equilibrium point in Section 4. Finally, in Section 5, an example is given to illustrate our results.

2. Preliminaries

Let $C[X, Y]$ be a continuous mapping set from the topological space X to the topological space Y, and $R_+ = [0, +\infty)$. Especially, $C \triangleq C[(-\infty, 0], R^n]$.

We consider Hopfield neural networks with distributed delays as follows:

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_{ij}(t-s) g_j(x_j(s)) \mathrm{d}s + I_i, & t \geqslant 0, \\ x_i(t) = \varphi_i(t), & -\infty < t \leqslant 0, \quad i = 1, \ldots, n, \end{cases} \tag{2.1}$$

where $c_i > 0$ represent the passive delay rates, $i = 1, \ldots, n$, in which n corresponds to the number of units in the networks, $x_i(t)$ corresponds to the state vectors of the ith neural unit at time t, a_{ij}, b_{ij} are the synaptic connection strengths, f_i, g_i are the signal propagation functions, I_i is the constant input from outside the system, φ_i is assumed to be bounded and continuous functions on $(-\infty, 0]$. The kernel functions $k_{ij} : [0, +\infty) \to [0, +\infty)$ $(i, j = 1, \ldots, n)$ are continuous on $[0, +\infty)$ with $\int_0^\infty k_{ij}(s)\mathrm{d}s = 1$.

For convenience, in the following, we shall rewrite Eq. (2.1) in the form: