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Flow separation behind ellipses at Reynolds numbers less than 10

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ABSTRACT

Flow separation behind two-dimensional ellipses with aspect ratios ranging from 0, a flat plate, to 1, a circular cylinder, were investigated for Reynolds numbers less than 10 using both a cellular automata model and a commercial computational fluid dynamics software program. The relationship between the critical aspect ratio for flow separation and Reynolds number was determined to be linear for Reynolds numbers greater than one. At slower velocities, the critical aspect ratio decreases more quickly as the Reynolds number approaches zero. The critical Reynolds numbers estimated for flow separation behind a flat plate and circular cylinder agree with extrapolations from experimental observations. Fluctuations in the values of the stream function for laminar flow behind the ellipses were found at combinations of Reynolds number and aspect ratio near the critical values for separation.

1. Introduction

For Reynolds numbers between 1 and 10, smooth flows around circular cylinders are likely to remain attached whereas flows around flat plates oriented normal to the flow are separated and exhibit recirculating downstream vortices. It is therefore reasonable to expect that flows around nearly circular ellipses are likely to remain attached, whereas flows around narrower ellipses will exhibit separation and recirculating vortices. By implication, for a given Reynolds number, there should be a critical aspect ratio, defined as the length of the ellipse axis aligned parallel to the mean flow divided by the length of the crosswise axis, at which flow separation occurs. The critical aspect ratio is expected to vary as a function of Reynolds number. Similarly, for an ellipse of given aspect ratio there should be a critical Reynolds number for the onset of flow separation.

These investigations examined the onset of flow separation behind two-dimensional ellipses with aspect ratios ranging from 0, a flat plate, to 1, a circular cylinder at Reynolds numbers less than 10. For Reynolds numbers greater than 1, a linear relationship was found between the Reynolds number and the critical aspect ratio for separation. For slower flows, the critical aspect ratio decreases more quickly as the Reynolds number goes to 0. The critical Reynolds numbers for separation behind a cylinder and a flat plate agree with extrapolations from experimental observations. Fluctuations in both the attached and separated laminar flow behind ellipses were found at combinations of Reynolds number and aspect ratio that were near the critical values for separation.

In this paper we review the literature pertaining to low Reynolds number flow around two-dimensional ellipses and the previous applications of the FHP-I cellular automata model to such flows. Next, we present our application of the model to ellipses of various aspect ratios and compare the determinations of attached and separated flow to those obtained with the FLUENT computational fluid dynamics software program. Finally, the results of both models are compared to experimental data.

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2. Slow flow around elliptical cylinders

The flow around circular cylinders has been studied with experimental, analytical and numerical techniques [1]. However, there is a paucity of experimental and computational data for flow around ellipses at low Reynolds numbers that is due, at least in part, to the economic imperatives for studying higher Reynolds number flows that are common for vehicles and machinery. However, microfluidics research is now becoming recognized as a field in its own right [2].

When using inviscid models, the flow pattern is symmetrical not only above and below the cylinder, but also upstream and downstream. However, for viscous flows above Reynolds numbers of approximately one, dynamic forces override the fore and aft symmetry. As the flow rate increases, a separation appears on the downstream side that contains two recirculating eddies that grow laterally as the Reynolds number increases.

In the mid 19th century, Stokes developed an analytical description of flow past a motionless sphere at Reynolds numbers less than 1. Oseen extended Stokes's work to cylinders using an analysis based upon slight deviations from a known flow [3].

Schlichting reported on the work by Blasius and others in the early 1900s to develop exact solutions for flow around simple shapes [4]. The velocity of the potential flow and the velocity profile of the boundary layer were expressed as power series in x, the distance from the stagnation point measured along the object's contour. Lack of sufficient computational resources to include an adequate number of terms limited the accuracy of the calculations, especially for slender body shapes such as streamlined ellipses.

Taneda experimented with circular cylinders and flat plates aligned parallel to the flow for Reynolds numbers in the range 1–2000 [5]. He observed the formation of twin rear vortices behind a circular cylinder at a Reynolds number of 7. He later extended his work by taking detailed measurements of the recirculating eddies behind flat plates oriented normal to the flow [6]. He found measurable eddies at a Reynolds number of 0.92. He also measured the relationship between the eddy size and Reynolds number.

Dennis and Chang presented finite difference solutions of the equations of motion for steady, incompressible flow around a circular cylinder for Reynolds numbers in the range 5–100 [7]. Like Taneda [5], they found a linear growth in eddy length with increasing Reynolds number behind a circular cylinder. They calculated that flow separation begins at a critical Reynolds number of 6.2.

Nieuwstadt and Keller modeled viscous flow around a circular cylinder for Reynolds numbers in the range 1–40 using the semi-analytical method of series truncation to express the stream function and vorticity in a Fourier series that was substituted into the Navier–Stokes equation to yield a finite system of nonlinear ordinary differential equations [8]. Their results compared favorably with Dennis and Chang [7] for Reynolds numbers less than 40, and were computationally more efficient.

Coutanceau and Bouard photographed features of wakes behind circular cylinders for Reynolds numbers in the range of 5–40 [1]. They noted that the maximum recirculating velocity on the axis between the eddies increased linearly with Reynolds number.

Van Dyke published experimental visualizations of flow around circular cylinders at Reynolds numbers of 0.16 and 1.54 [9]. In the former case, the flow was almost completely symmetrical upstream and downstream of the object. In the latter case, the streamlines downstream of the cylinder were elongated, but the flow was not separated. He also published Taneda's photograph [6] of flow past a flat plate normal to the flow at a Reynolds number of 0.334. Although Taneda did not claim flow separation for this case, Van Dyke entertained the possibility.

Shintani, Umemura and Takano asymptotically matched the Stokes and Oseen solutions of the Navier–Stokes equations for two overlapping regions near elliptic cylinders at a Reynolds number of 0.1 [10]. For a flat plate, two symmetrical recirculating vortices formed on the downstream side. They reported qualitative agreement with Taneda's illustration [6] of flow at a Reynolds number of 0.44.

Nakayama et al. presented visualizations of flows around a circular cylinder at Reynolds numbers of 0.038 and 1.1 [11]. In both cases the flow was attached but did not exhibit fore and aft symmetry.

Wu and Lee presented experimental data and mathematical calculations using the FIDAP computational fluid dynamics software program for the free settling of solid and porous ellipsoids of revolution for Reynolds numbers in the range 0.1–40 [12]. For a solid ellipsoid of revolution of aspect ratio 0.7 with the major axis aligned parallel to the flow, the upstream and downstream streamlines were symmetrical at a Reynolds number of 0.1. At a Reynolds number of 40, recirculating eddies were visible on the downstream side.

3. Previous cellular automata models and applications

Flow systems are usually modeled mathematically in terms of continuous functions in an Eulerian system. Fluids, however, are composed of discrete molecules. Although, it is not feasible to track every molecule, the field of cellular automata (CA) has demonstrated that it is possible to compute a large, yet tractable, number of automata that emulate fluid particles. The system as a whole can be considered a continuum so long as the length scale of the macroscopic motions is much larger than the length scale of the automata [13].

The first fully deterministic lattice gas model, known as HPP, is based upon unit-mass, unit-speed fluid particles moving either horizontally or vertically between nodes along a square lattice [14]. When only two particles experience a head-on collision at a node, they leave at right angles along the two previously unoccupied directions. Viscosity is anisotropic because