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2D Green's functions for semi-infinite orthotropic thermoelastic plane

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A B S T R A C T

Based on the 2D general solutions of orthotropic thermoelastic material, the Green's function for a steady point heat source in the interior of semi-infinite orthotropic thermoelastic plane is constructed by three newly introduced harmonic functions. All components of coupled field in semi-infinite thermoelastic plane are expressed in terms of elementary functions. Numerical results are given graphically by contours.

1. Introduction

Fundamental solutions or Green's functions play an important role in both applied and theoretical studies on the physics of solids. They are foundations of a lot of further works. For example, fundamental solutions can be used to construct many analytical solutions of practical problems when boundary conditions are imposed. They are essential in the boundary element method as well as the study of cracks, defects and inclusions.

For isotropic materials, there is well-known closed-form Kelvin Green's function [1]. For transversely isotropic materials, Lifshitz and Rozentsveig [2] and Lejcek [3] presented the Green's functions using the Fourier transform method. Elliott [4], Kroner [5] and Willis [6] obtained the Green's functions using the direct method. Sveklo [7] obtained Green's functions using the complex method. Pan and Chou [8] and Ding et al. [9] presented the Green's function in form of compact elementary functions. For anisotropic materials, Pan and Yuan [10] and Pan [11] obtained the three-dimensional Green's functions for bimaterials with perfect and imperfect interfaces, respectively. The thermal effects are not considered in all above works.

Sharma [12] gave the fundamental solution of transversely isotropic thermoelastic materials in an integral form. Yu et al. [13] gave the Green's function for a point heat source in two-phase isotropic thermoelastic materials. Chen et al. [14] derived a compact 3D general solution for transversely isotropic thermoelastic materials. In this general solution, all components of thermoelastic field are expressed by three harmonic functions.

In this paper, 2D Green's functions for a steady point heat source in a semi-infinite orthotropic thermoelastic plane is investigated. For this object, the 2D general solution, which is parallel to 3D general solution of Chen et al. [14], is presented in Section 2. In Section 3, three new suitable harmonic functions are constructed in form of elementary functions with undetermined constants by the method of trial-and-error. The corresponding coupled field can be obtained by substituting these functions into the general solution, and the undetermined constants can be obtained by the continuous conditions on plane

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$z = h$, equilibrium conditions of a rectangle within $0 < a_1 < z < a_2$ and $-b \leqslant x \leqslant b$ and boundary conditions on surface $z = 0$. Numerical examples are presented in Section 4. Finally, the paper is concluded in Section 5.

2. 2D General solution for orthotropic thermoelastic material

The 3D equations of orthotropic thermoelastic materials can be found in Kumara and Singh. [15]. If all components are independent of coordinate y, one can have the so-called plane problem. The constitutive equations in 2D Cartesian coordinates (x, z) can be simplified as

$$\begin{aligned}\sigma_x &= c_{11}\frac{\partial u}{\partial x} + c_{13}\frac{\partial w}{\partial z} - \lambda_{11}\theta,\\ \sigma_z &= c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial w}{\partial z} - \lambda_{33}\theta,\\ \tau_{zx} &= c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),\end{aligned} \tag{1}$$

where u and w are components of the mechanical displacement in x and z directions, respectively; σ_{ij} are the components of stress; θ is temperature increment, respectively; c_{ij} and λ_{ii} are elastic constants and thermal moduli, respectively.

In the absence of body forces, the mechanical and heat equilibrium equations are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0, \quad \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \sigma_z}{\partial z} = 0, \tag{2a}$$

$$\left(\beta_{11}\frac{\partial^2}{\partial^2 x} + \beta_{33}\frac{\partial^2}{\partial^2 z}\right)\theta = 0, \tag{2b}$$

where $\beta_{ii}(i = 1,3)$ are coefficients of heat conduction.

By virtue of the parallel method of Chen et al. [14], 2D general solution for Eqs. (1) and (2) can be obtained as follows:

$$u = \sum_{j=1}^{3}\frac{\partial \psi_j}{\partial x}, \quad w = \sum_{j=1}^{3} s_j k_{1j}\frac{\partial \psi_j}{\partial z_j}, \quad \theta = k_{23}\frac{\partial^2 \psi_3}{\partial z_3^2}, \tag{3a}$$

$$\sigma_x = -\sum_{j=1}^{3} s_j^2 \omega_j \frac{\partial^2 \psi_j}{\partial z_j^2}, \quad \sigma_z = \sum_{j=1}^{3}\omega_j\frac{\partial^2 \psi_j}{\partial z_j^2}, \quad \tau_{zx} = \sum_{j=1}^{3} s_j\omega_j\frac{\partial^2 \psi_j}{\partial x \partial z_j}, \tag{3b}$$

where

$$z_j = s_j z \quad (j = 1,2,3). \tag{4}$$

$s_3 = \sqrt{\beta_{11}/\beta_{33}}$, $s_j(j = 1,2)$ satisfying $Re(s_j) > 0$ are the two eigenvalues of the fourth degree polynomial Eq. (11) in Chen et al. [14]. Functions $\psi_j(j = 1, 2, 3)$ satisfy, respectively, the following harmonic equations:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2}\right)\psi_j = 0 \quad (j = 1,2,3), \tag{5}$$

and

$$k_{1j} = \alpha_{j1}/s_j, \quad k_{2j} = \alpha_{j2}, \tag{6a}$$

$$\omega_j = [c_{11} - c_{13}k_{1j}s_j^2 + \lambda_{11}k_{2j}]/s_j^2 = c_{44}(1 + k_{1j}) = -c_{13} + c_{33}k_{1j}s_j^2 - \lambda_{33}k_{2j}, \tag{6b}$$

where α_{jm} $(m = 1,2)$ are constants defined in Eq. (18) of Chen et al. [14]. It should be noted that the general solutions given in Eq. (3) are only valid for the case when the eigenvalues s_j $(j = 1, 2, 3)$ are distinct, which is the most common case.

3. Green's functions for a point heat source in the interior of a semi-infinite orthotropic thermoelastic plane

Consider a semi-infinite orthotropic thermoelastic plane $z \geqslant 0$ (Fig. 1). A point heat source of strength H is applied at the point $(0, h)$ in 2D Cartesian coordinate (x,z). The surface $(z = 0)$ is free and thermally insulated. Based on the general solution Eq. (3), the coupled field in the semi-infinite thermoelastic plane is derived in this section.

The boundary conditions on the surface $(z = 0)$ are in the form of

$$\sigma_z = \tau_{zr} = 0, \quad \partial\theta/\partial z = 0. \tag{7}$$

For future reference, following denotations are introduced:

$$\begin{aligned} &z_j = s_j z, \quad h_k = s_k h,\\ &z_{jk} = z_j + h_k, \quad r_{jk} = \sqrt{x^2 + z_{jk}^2},\\ &\bar{z}_{jk} = z_j - h_k, \quad \bar{r}_{jk} = \sqrt{x^2 + \bar{z}_{jk}}, \quad (j,k = 1,2,3).\end{aligned} \tag{8}$$