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Modelling of thermoelastic Rayleigh waves in a solid underlying a fluid layer with varying temperature

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ABSTRACT

The present paper is aimed at to study the propagation of surface waves in a homogeneous isotropic, thermally conducting and elastic solid underlying a layer of viscous liquid with finite thickness in the context of generalized theories of thermoelasticity. The secular equations for non-leaky Rayleigh waves, in compact form are derived after developing the mathematical model. The amplitude ratios of displacements and temperature change in both media at the surface (interface) are also obtained. The liquid layer has successfully been modeled as thermal load in addition to normal (hydrostatic pressure) one, which is the distinctive feature of the present study and missing in earlier researches. Finally, the numerical solution is carried out for aluminum-epoxy composite material solid (half-space) underlying a viscous liquid layer of finite thickness. The computer simulated results for dispersion curves, attenuation coefficient profiles, amplitude ratios of surface displacements and temperature change have been presented graphically, in order to illustrate and compare the theoretical results. The present analysis can be utilized in electronics and navigation applications in addition to surface acoustic wave (SAW) devices.

1. Introduction

The classical theory of heat conduction predicts an infinite speed of heat transportation, which contradicts the physical facts. Non-classical theories have been developed to alleviate this paradox. A flux-rate term in Fourier's law of heat conduction in order to formulate a generalized theory that admits finite speed for thermal signals was incorporated by Lord and Shulman [1]. Green and Lindsay [2] included a temperature rate among the constitutive variables to develop a temperature-rate-dependent thermoelasticity that does not violate the classical Fourier's law of heat conduction when the body under consideration has a center of symmetry; this theory also predicts a finite speed of heat propagation. According to these theories, heat propagation should be viewed as a wave phenomenon rather than diffusion one. A wave-like thermal disturbance is referred to as 'second sound' by Chandrasekharaiah [3]. It was experimentally proved by many researchers [4–6] for solid helium that thermal waves (second sound) propagating with finite, though quite large, speed also exists. Nayfeh and Nasser [7] studied the propagation of plane harmonic and Rayleigh waves in thermoelastic solids in the context of generalized thermoelasticity developed by Lord and Shulman [1]. The propagation of Lamb waves in a plate bordered with inviscid liquid layers on both sides was studied by Wu and Zhu [8]. Zhu and Wu [9] investigated the propagation of Lamb waves in a plate bordered by viscous liquid and also presented numerical solution of the Lamb wave dispersion relations related to sensing applications. Nayfeh and Nagy [10] derived the exact characteristic equations for leaky waves propagating along the interfaces of several systems involving isotropic elastic solids loaded with viscous fluids including half-spaces and finite

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thickness plates totally immersed in fluids or coated on one or on both sides by finite thickness fluid layers. The technique adopted by Nayfeh and Nagy [10] removed certain inconsistencies that unnecessarily reduce the accuracy and range of validity of Zhu and Wu [9] results. The influence of viscous fluid loading on the propagation of leaky Rayleigh wave in the presence of heat conduction effects was studied by Qi [11]. Sharma and Pathania [12,13] studied the propagation of Rayleigh-Lamb waves in homogeneous isotropic plates bordered with layers of inviscid liquid on both sides in the context of different generalized theories of thermoelasticity.

Achenbach [14] says unlike the hyperbolic solutions, the classical solutions show no distinct wave front and therefore as expected and increase in temperature starts at the initial time. However, the difference in the predicted temperature between the two theories is small and only apparent for very small time scale (of the order of 100 picoseconds). These time scales are large enough for the solutions given by both the theories to be numerically undistinguishable in case of many non-destructive evaluation (NDE) applications. Thus, for convenience, the theory can be selected for the time scales of interest with no practical effect on calculated results. Similarly, the choice of a specific value for the heat propagation speed in the hyperbolic equation does not affect the results, however its value chosen equal to the speed of longitudinal waves in the hyperbolic formulation, presents some numerical advantages from the practical point of view. Further, overlying liquid with varying temperature can be utilized as a thermal source in addition to normal load (hydrostatic pressure) simultaneously in many engineering applications. This distinctive feature has not been studied in the earlier researches as per knowledge of the authors.

Keeping all these points in view, the hyperbolic model of governing equations has been employed to investigate the present problem. We analyze the Rayleigh wave propagation in thermoelastic solids underlying a viscous layer of finite thickness. More general dispersion equations of non-leaky thermoelastic Rayleigh type waves are derived. The conventional coupled thermoelasticity (CT) and generalized theories of thermoelasticity namely, Lord–Shulman (LS) and Green–Lindsay (GL) have been employed to study the problem and to investigate the effect of thermal relaxation times. In the absence of viscosity, the analysis reduces to that of Sharma and Pathania [12] for inviscid liquid loading maintained at uniform temperature. Numerical solution of the dispersion equations, displacement magnitudes and temperature change for an aluminum-epoxy composite half-space cladded with a viscous liquid layer of finite thickness has also been carried out and presented graphically.

2. Formulation of the problem and formal solution

We consider a viscous liquid layer of finite thickness d overlying a homogeneous isotropic, thermally conducting elastic solid in the undeformed state at uniform temperature T_0. The origin of the coordinate system (x,y,z) is taken at any point on the plane surface (interface) and z-axis points vertically downward into the solid half-space which is thus represented by $z \geqslant 0$. We choose x-axis in the direction of wave propagation in such a way that all the particles on a line parallel to y-axis are equally displaced. Therefore, all the field parameters become independent of y-coordinate. Further, it is assumed that the disturbances are small and are confined to neighbourhood of the interface $z = 0$ and hence vanish as $z \to \infty$ (see Fig. 1).

The basic governing field equations of motion and heat conduction, in the absence of heat sources and body forces in the non-dimensional form, are given by [12]

$$\delta^2\nabla^2\vec{u} + (1-\delta^2)\nabla\nabla\cdot\vec{u} - \nabla(T + t_1\delta_{2k}\dot{T}) = \ddot{\vec{u}} \tag{1}$$

$$\nabla^2 T - (\dot{T} + t_0\ddot{T}) = \varepsilon\nabla\cdot(\dot{\vec{u}} + t_0\delta_{1k}\ddot{\vec{u}}) \tag{2}$$

where $\delta^2 = \frac{c_2^2}{c_1^2}, c_2^2 = \frac{\mu}{\rho}, c_1^2 = \frac{\lambda+2\mu}{\rho}, \varepsilon = \frac{\beta^2 T_0}{\rho C_e(\lambda+2\mu)}$. Here c_1, c_2 are respectively, the longitudinal and shear wave velocities in the thermoelastic half-space; λ, μ are Lame's parameters; ε is the thermomechanical coupling constant; $T(x,z,t)$ is temperature change and $\vec{u}(x,z,t) = (u,0,w)$ is the displacement vector; δ_{jk} is the Kronecker delta with $k = 1$ for LS theory and $k = 2$ for GL theory; t_0 and t_1 are non-dimensional thermal relaxation times; ∇^2 is Laplacian operator and superposed dot is used for time differentiation.

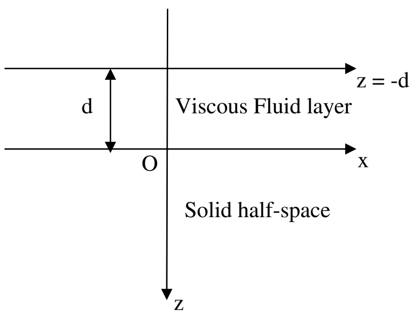

Fig. 1. Geometry of the problem.