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Numerical model for vibration damping resulting from the first-order phase transformations

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Abstract

A numerical model is constructed for modelling macroscale damping effects induced by the first-order martensite phase transformations in a shape memory alloy rod. The model is constructed on the basis of the modified Landau–Ginzburg theory that couples nonlinear mechanical and thermal fields. The free energy function for the model is constructed as a double well function at low temperature, such that the external energy can be absorbed during the phase transformation and converted into thermal form. The Chebyshev spectral methods are employed together with backward differentiation for the numerical analysis of the problem. Computational experiments performed for different vibration energies demonstrate the importance of taking into account damping effects induced by phase transformations.

Keywords: Martensite transformation; Thermo-mechanical coupling; Vibration damping; Ginzburg–Landau theory

1. Introduction

Shape memory alloy (SMA) materials are able to directly transduce thermal energy into mechanical and vice versa. Their unique properties make them very attractive in many engineering applications, including mechanical and control engineering, biomedicine, communication, robotics and so on [1]. Among these applications, dampers made from SMA for passive and semi-active vibration damping, are quoted perhaps most frequently [2]. These devices exhibit fairly complicated nonlinear (hysteretic) behaviour induced by martensite phase transformations. An appropriate mathematical model involving phase transformations and thermo-mechanical coupling is essential for a better understanding of the dynamic behaviour of SMA dampers.

The damping effects of SMA cannot be understood without incorporating into the model the *dynamics* of the first-order martensite transformations induced by mechanical loadings, and hysteresis as a consequence. SMAs have more than one martensite variants at low temperature that correspond to the same elastic

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potential energy of the material stored via deformation. When the martensite phase transformation is induced mechanically, part of the external energy will be consumed to transform the material from one variant to another, without increasing the total elastic energy stored in the material, but the deformation will be oriented in a way favored by the external loadings. The work done by the external loadings during the phase transformation process cannot be restored reversely when the loading is removed, since it is converted into thermal form, and will be dissipated eventually via, for example, thermal dissipation. This dissipation of mechanical energy due to phase transformation can be measured by the hysteretic behaviour of the material under mechanical loadings [3,4]. Because of the nonlinear nature of phase transformations and the nonlinear coupling between the mechanical and thermal fields, the dynamic behaviour of SMA dampers becomes very complex.

Some investigations have been already carried out to understand the damping effects of SMA at either micro-scale or macro-scale [4–6]. Among many efforts, it was shown in Refs. [4,7] (and references therein) that the damping effects of SMA are influenced by the vibration frequency, heating/cooling rate (temperature change rate), and vibration amplitude. This influence is discussed and analyzed in terms of strain formation, interface moving, internal friction, and other factors, which are all considered either at micro-scale or meso-scale.

For most current engineering applications of SMA dampers, damping effects due to hysteresis is more pronounced at macroscale, and one needs to model the dynamical behaviour and damping effects of SMAs at macroscale, which demands that the model should have the capability to capture all the contributions to vibration damping, particularly that due to hysteresis induced by cyclic mechanical loadings [3,7]. Since vibration damping can be induced by hysteretic behaviour of the SMAs, interface moving, internal friction, and thermo-mechanical coupling, the model for damping devices made from SMA has to be constructed on the basis of dynamics of the materials, which involves the phase transformation and thermo-mechanical coupling. Progressively, the impact induced phase transformation in SMA wires were investigated in [6,7] where the constitutive laws were approximated linearly, and the thermo-mechanical coupling was neglected. Such models have obvious limitations.

In what follows, we attempt to overcome such limitations by better capturing the thermo-mechanical coupling, hysteresis, and nonlinear nature of phase transformations with the Ginzburg–Landau theory, originally discussed in the context of SMAs in [8,9] and later applied to model the dynamical behaviour of SMA rods under dynamical loadings (e.g., [10–12] and references therein). In particular, in this paper we employ a mathematical model based on the modified Ginzburg–Landau theory for the SMA damper, the dynamics of the SMA damper are described by Navier's equation with a non-convex constitutive relation. The damper is connected with a mass block with a given initial velocity. The movement of the mass block is then simulated as a single degree of freedom system, subject to the damping force from the SMA rod.

2. Mathematical modelling

The physical problem investigated here is sketched in Fig. 1. There is a mass block connected to a SMA rod. The SMA rod has a cross section area of β and length L, and is initially at rest. The block has an initial velocity v_m. The purpose of the current investigation is to show the damping effect of the SMA rod on the vibration of the mass block.

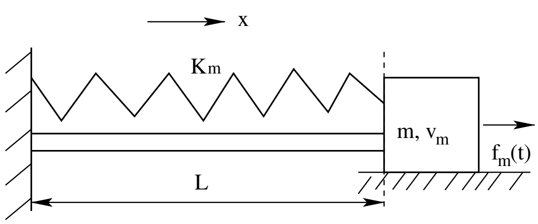

Fig. 1. Vibration damping of a mass block connected to a shape memory alloy rod.