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Stress analysis of orthotropic planes weakened by cracks

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Abstract

The stress fields in an orthotropic infinite plane containing Volterra type climb and glide edge dislocations are derived. The dislocation solutions are utilized to formulate integral equations for dislocation density functions on the surfaces of smooth cracks. The integral equations are of Cauchy singular type and are solved for several different cases of crack configurations and arrangements. The results are used to evaluate modes I and II stress intensity factors for multiple smooth cracks.

Keywords: Edge dislocation; Dislocation density; Smooth crack; Stress intensity factor

1. Introduction

Composite materials are indispensable in many industries, necessitating the thorough comprehension of their behavior under load. The cracking of orthotropic materials occurs during manufacturing or service life of a mechanical component and may be considered as the major cause of failure. The stress analysis of a cracked body is the prerequisite for damage evaluation.

The earliest investigation concerning an orthotropic plane weakened by a crack was achieved by Stroh [1]. In his article, the crack was parallel to a principal axis of orthotropy. Further progress in the problem may be enumerated as follows. Ang and Williams [2] employed integral equation method to analyze an orthotropic plate having a finite crack. Sih et al. [3] used complex variable technique and obtained general solution for stress fields in the vicinity of crack tip in an anisotropic body. Binienda et al. [4] treated a bent crack in an infinite plane under far-field uniform tensile traction. The bent crack was modeled by two approaching

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Nomenclature

a, b lengths of major and minor axes of elliptical crack
A_{ij} coefficients matrix
$A(\omega), B(\omega)$ unknowns coefficients
b_s, b_n dislocation densities in orthogonal coordinates s, n
b_x, b_y dislocation densities
B_x, B_y components of Burgers vector
E_x, E_y, G_{xy} elastic moduli
$g_s(t), g_n(t)$ regular terms of dislocation densities in orthogonal coordinates s, n
$H(x)$ Heaviside step function
$k_{\mathrm{IL}}, k_{\mathrm{IIL}}, k_{\mathrm{IR}}, k_{\mathrm{IIR}}$ stress intensity factors of left and right side of crack
k_0 stress intensity factor of a crack in infinite plane
$k_{11ij}(\eta, t), k_{12ij}(\eta, t), k_{21ij}(\eta, t), k_{22ij}(\eta, t)$ kernels of integral equations
l half-length of crack
N number of cracks
r_1, r_2 constants designating material orthotropy
u, v displacement components
u_s, u_n displacement components in orthogonal coordinates s, n
x, y coordinates
$\alpha(t), \beta(t)$ parametric equations of the crack
$\delta(\omega)$ Dirac delta function
δ_{ij} Kronecker delta
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$ strain components
θ crack orientation
κ Kolosov constant
v_{xy} Poisson's ratio
σ_n, σ_s stress components in orthogonal coordinates s, n
$\sigma_x, \sigma_y, \sigma_{xy}$ stress components
$\sigma_{x\infty}, \sigma_{y\infty}, \tau_{xy\infty}$ applied tractions at infinity
ϕ Airy stress function
$\psi_i(t)$ angle between s- and x-axes

straight cracks which eventually intersect. They formulated the problem in terms of integral equations with generalized Cauchy kernels and the stress intensity factors at the crack tips were evaluated by numerical solution of the resulting system of equations. Mauge and Kachanov [5] obtained the stress fields in a composite medium with a crack by taking the limit of the Savin's solution [6] for an arbitrary oriented elliptical hole. The solution was then used for the determination of stress intensity factors in an orthotropic plane weakened by several interacting straight cracks. The interaction of two arbitrarily oriented straight cracks in a homogenous general anisotropic medium was the subject of investigation by Peng and Sung [7]. The solution of a concentrated dislocation interacting with a straight crack was employed to obtain integral equations for cracks. Nonetheless, the procedure becomes extremely involved for curved and multiple interacting straight cracks.

In this article, an elementary procedure is devised for the analysis of multiple curved cracks in orthotropic planes under in-plane loads. The stress fields in an infinite orthotropic plane caused by a climb and/or glide Volterra type dislocation are obtained in closed forms. For an isotropic plane the dislocation solutions recover the well-known results in literature. The stress fields due to dislocations are then used to derive singular integral equations for a plane with multiple cracks under tensile and shear traction. In the particular case of an