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ScienceDirect
Applied Mathematical Modelling 31 (2007) 2684–2695
APPLIED MATHEMATICAL MODELLING
www.elsevier.com/locate/apm

A mathematical description of the acoustic coupling of the mass/spring model

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Received 1 September 2005; accepted 10 October 2006
Available online 12 December 2006

Abstract

This paper describes hybrid mathematical model which couples the mechanics of the mass/spring model to the acoustic wave propagation model for use in generating the acoustic signal emitted by complex structures of paper fibres under strain. A discussion of the coupling method is presented including remarks on the errors encountered intrinsic to the discretisation scheme. The numerical results of a vibrating rubber band and a vibrating paper fibre are compared to their experimental counterparts. The fundamental frequencies of the acoustic signals are compared showing a close agreement between the experimental and numerical results.

Keywords: Paper; Acoustic emission; Lumped systems; Acoustic wave equation

1. Introduction

The work presented in this contribution is part of a larger project [1], which aims to investigate the use of the acoustic emission (AE) monitoring technique in the prediction and identification of the damage mechanisms in paper, associated with its production process. The two damage mechanisms proposed in [1] are a fibre/fibre bond failure and a fibre failure. To achieve this goal, it was clear that a numerical model which can generate an AE from the movement of the microscopic structure of paper was required. The model techniques used must be able to satisfy the following criteria.

- The ability to model the movement of a complex structure of fibres.
- The ability to handle the dynamic changes in the fibre structure caused by the damage mechanisms.
- The ability to generate an AE from the resulting movement.

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doi:10.1016/j.apm.2006.10.016

Current numerical schemes for vibro-acoustic systems are almost exclusively based on Finite (FEM) and Boundary (BEM) Element Methods as they provide easy coupling of the vibro-acoustic equations. There are numerous resources available on this subject [2–4], along with several commercial offerings [5–7]. The FEM and BEM model the vibration of a continuous surface and couple this movement to the acoustic wave equation to generate the acoustic radiation. In contrast, this work requires the movement of the paper structure to be simulated from a microscopic level at which there is no continuous surface to model; instead the surface comprises a number of interconnected fibres as seen in Fig. 8. Therefore element methods cannot be applied to this problem.

It has been decided to approximate the paper fibres to a series of masses and springs, a technique known as the mass/spring or lumped system. The mass/spring model has seen extensive use in the area of real-time animation of cloth [8–10] and hair [11,12] and is an ideal candidate for modelling the movement of the paper fibre structure. There are software packages available that use the mass/spring paradigm, both free [13] and commercial [14,15], but they do not provide the flexibility needed to model the damage mechanisms. However, by manipulating the equation of motion of the mass/spring model it is possible to simulate the damage mechanisms, which will be presented in a later paper.

To generate the AE from the moving fibre structure, the acoustic wave equation will be solved using a Finite Difference Time Domain (FDTD) method [16]. The pressure source terms for the acoustic wave are calculated directly from the mass/spring model and provide the coupling in the heterogeneous system. This report will provide a detailed derivation of both the mass/spring and acoustic wave models followed by a discussion of the problems encountered when coupling the two models together.

The accuracy of the coupled model is then compared against experimental data from two types of material. The first experimental data set is obtained from a rubber band stretched between two points and plucked. The rubber band is used as it can be thought of as a macroscopic 'fibre' with well known material properties. The second experimental data set is obtained by stretching a 10 mm × 100 mm paper sample until fracture occurs under laboratory conditions.

The following sections will include a derivation of the hybrid model including remarks on the intrinsic errors generated when coupling the two models together, followed by a section verifying the accuracy of the hybrid model, finishing with a discussion on the future work the model will provide.

2. Mass/spring (lumped) model

Consider the mass/spring model unit cell (see Fig. 1): where x is the displacement of the mass, m is the mass's mass, F_f is the friction force on the mass, k is the stiffness of the spring, E is the Young's modulus of the spring, t_s is the thickness of the spring, w_s is the width of the spring, and L is the equilibrium length of the spring.

The force needed to displace a mass m by a distance x can be calculated using Newton's Second Law.

$$F_m = m\frac{d^2x}{dt^2}. \tag{2.1}$$

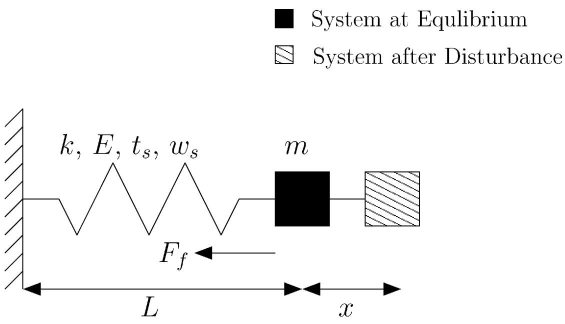

Fig. 1. Mass/spring model unit cell.