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Finite element and boundary element contact stress analysis with remeshing technique

Cuneyt Oysu

Kocaeli University, Engineering Faculty, Department of Mechatronics Engineering, Veziroglu Campus, Izmit 41040, Turkey

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Abstract

The fundamental part of the contact stress problem solution using a finite element method is to locate possible contact areas reliably and efficiently. In this research, a remeshing technique is introduced to determine the contact region in a given accuracy. In the proposed iterative method, the meshes near the contact surface are modified so that the edge of the contact region is also an element's edge. This approach overcomes the problem of surface representation at the transition point from contact to non-contact region. The remeshing technique is efficiently employed to adapt the mesh for more precise representation of the contact region. The method is applied to both finite element and boundary element methods. Overlapping of the meshes in the contact region is prevented by the inclusion of displacement and force constraints using the Lagrange multipliers technique. Since the method modifies the mesh only on the contacting and neighbouring region, the solution to the matrix system is very close to the previous one in each iteration. Both direct and iterative solver performances on BEM and FEM analyses are also investigated for the proposed incremental technique. The biconjugate gradient method and LU with Cholesky decomposition are used for solving the equation systems. Two numerical examples whose analytical solutions exist are used to illustrate the advantages of the proposed method. They show a significant improvement in accuracy compared to the solutions with fixed meshes.

Keywords: Contact; Finite element; Boundary element; Mesh refinement; Remeshing; Lagrange multipliers

1. Introduction

The analysis of contact problems is a major concern in many engineering applications such as ball bearings, gears, rollers and pressure vessel attachments. Many practical engineering problems involve the contact of bodies with non-conforming contact situations, and are loaded by tangential as well as normal forces. The numerical modelling of practical contact problems requires special attention because the actual contact area between the contacting bodies is usually not known in advance. The correct solution of contact problems must be determined by an iterative scheme. This study covers a range of Hertzian contact problems without friction and tangential loading.

E-mail address: coysu@kou.edu.tr

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The rapid progress of the finite element method in the early seventies led to its application to contact problems. Wilson and Parson [11] were the first to apply the finite element (FEM) formulation to two-dimensional frictionless contact problems using constant strain elements, while Francavilla and Zienkiewicz [1] used parabolic elements. The range of applications was later extended to include friction and non-linear problems. Contact conditions can be imposed on a set of finite element equations mostly using penalty methods with gap elements or Lagrange multipliers method [2,3] or mixed computational algorithms [4].

In the analysis of most non-conforming bodies, the contact surfaces are continuous and may be represented initially before the application of contact constraints by second degree polynomials. When the external loading is applied, the contact region advances as a function of the relative approach of the contacting bodies. In the analysis of advancing contact problems to obtain a correct solution, the very narrow region between the contacting and the free surface of deformed body should be represented with higher degree polynomials. Alternatively, the accuracy can be improved when the transition point is between two adjacent elements, since the polynomial continuity is not necessary between the elements. During the solution iterations adaptation of mesh to move the transition point to the corner between two elements improves the solution. The incremental remeshing technique is used to modify the mesh between the contacting and non-contacting regions. The multigrid method introduced for contact stress problems by Hasanov [5] using the penalty function method was implemented for FEM. His incremental technique is modified to include Lagrange multiplier constraints for contact stress analysis.

The boundary element method (BEM) offers significant advantages over the FEM. Using the BEM the dimensionality of the problem is reduced by one and only the boundary has to be discretized. This reduces the computational time and data preparation labour considerably. The BEM is very suitable for contact problems as the non-linearity occur at the boundary. The ease of mesh preparation makes remeshing local regions much easier than in FEM meshes. The remeshing technique is then employed successfully for BEM analysis of contact problems. Since the method modifies the mesh only on the contacting and neighbouring region, the solution to the matrix system is very close to the previous one in each incrementation. Thus, an iterative solver namely bi-conjugate gradient [6] method is applied to decrease the total computation time. Both direct and iterative solver performances on BEM and FEM analyses are also investigated for the technique. The bi-conjugate gradient and LU with Cholesky decomposition are used for solving the equation systems. The method is applied to two well known contact problems with analytical solutions to verify the efficiency. Since the codes are written for two dimensional elastic frictionless contact stress analysis, a plane strain analysis of two cylinders in contact and axisymmetric analysis of a sphere compressed on rigid surface are selected for benchmarking.

2. Contact constraints for finite element method

The Lagrange Multiplier contact constraints method developed by Chaudhary and Bathe [7] is applied as the contact analysis technique. The contact of two bodies, namely target and contactor, shown in Fig. 1, is

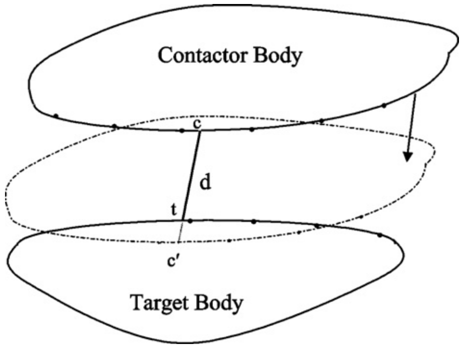

Fig. 1. Contactor body approaching the target body.