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A bulk arrival queueing model with fuzzy parameters and varying batch sizes

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Abstract

This paper develops a nonlinear programming approach to derive the membership functions of the steady-state performance measures in bulk arrival queueing systems with varying batch sizes, in that the arrival rate and service rate are fuzzy numbers. The basic idea is based on Zadeh's extension principle. Two pairs of mixed integer nonlinear programs (MINLP) with binary variables are formulated to calculate the upper and lower bounds of the system performance measure at possibility level α. From different values of α, the membership function of the system performance measure is constructed. For practice use, the defuzzification of performance measures is also provided via Yager ranking index. To demonstrate the validity of the proposed method, a numerical example is solved successfully.

Keywords: Bulk arrival; Queue; Fuzzy sets; Mixed integer nonlinear programming

1. Introduction

Bulk arrival queueing models have been widely applied to many practical situations, such as production/manufacturing systems, communication systems, and computer networks [1,2]. For example, when the operation in a production/manufacturing system will not begin until a

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specified number of raw materials are accumulated during an idle period, we often analyze this system by a bulk arrival queueing model that provides a powerful tool for evaluating the system performance.

In crisp environments, many articles on this topic have been published (for example, [3–7]). Most of the related studies are based on traditional queueing theory, in that the interarrival times and service times are assumed to follow certain probability distributions. However, in practice there are cases that these parameters may be obtained subjectively [8]. Thus, fuzzy queues are much more realistic than the commonly used crisp queues [8–10]. Several researchers have discussed fuzzy queueing systems, e.g., [8–14]; however, few articles have been published on the fuzzy bulk arrival queue with varying batch size. If the usual crisp bulk arrival queues can be extended to fuzzy bulk arrival queues, bulk arrival queueing models would have wider applications.

Relatively few articles have been published on the topic of fuzzy queues. Based on Zadeh's extension principle [15–17] and fuzzy Markov chain [18], Li and Lee [12] proposed a general approach for analyzing fuzzy queues. A straightforward idea is to apply their method to the fuzzy bulk arrival queueing problem. However, Negi and Lee [13] commented that their approach is so complicated that generally unsuitable for computational purposes; it can hardly derive analytical results for other more complicated queueing systems [19]. Consequently, it is very difficult and unsuitable to apply Li and Lee's approach to fuzzy bulk arrival queues. Negi and Lee [13] proposed a procedure using two-variable simulation [11] and the α-cut concept to analyze fuzzy queues. Unfortunately, their approach only provided crisp solutions. Kao et al. [19] investigated four simple fuzzy queueing problems, namely, $M/F/1$, $F/M/1$, $F/F/1$, and $FM/FM/1$, where F denotes fuzzy time and FM denotes fuzzified exponential time. It seems that their approach can be applied to the fuzzy bulk arrival queues. However, in bulk arrival queueing systems, customers arrive according to a compound Poisson process with random arrival size [1,2], which results in the fuzzy bulk arrival queues are much more complicated than the above four fuzzy queues, the solution procedure for the fuzzy bulk arrival queues is unknown and deserves further investigation.

Clearly, when the arrival rate or the service rate is fuzzy, the system performance measure of the bulk arrival queue should be fuzzy as well; it should be described by a membership function. In this paper, we develop a solution procedure that is able to provide fuzzy performance measures for bulk arrival queues with varying batch sizes, fuzzified exponential arrival rate and service rate. The membership functions of performance measures can be derived completely. The basic idea is to apply Zadeh's extension principle [15–17]. Two pairs of mixed integer nonlinear programming (MINLP) models are formulated to calculate the lower and upper bounds of the α-cut of the system performance measure. The membership function of the system performance measure is derived analytically.

2. Fuzzy bulk arrival queues with varying batch sizes

Consider a queueing system in which customers arrive at a single-server facility in batches as a Poisson process with group arrival rate $\tilde{\lambda}$ of all batches, where $\tilde{\lambda}$ is a fuzzy number, and all service times are independent and identically distributed according to an exponential distribution with