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On the refined integral method for the one-phase Stefan problem with time-dependent boundary conditions

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Abstract

Refined integral heat balance is developed for Stefan problem with time-dependent temperature applied to exchange surface. The method is applied to phase change in the half-plane and ordinary differential equation is obtained for the solid/liquid interface. The results are compared to those obtained by heat balance integral, perturbation and numerical methods.

Keywords: Stefan problem; Moving boundary; Refined integral heat balance; Analytical method

1. Introduction

During the two last past decades many investigations have been conducted to study heat transfer problem occurring with phase-change process. The problem well known as the Stefan problem as well as moving boundary problem arises in many physical processes such as melting of ice, heat storage, aerodynamic ablation, growth of metallic crystals and freezing of food. Owing to the

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Nomenclature

c	specific heat
F	dimensionless boundary temperature
k	thermal conductivity
l	length
Ste	Stefan number
t	time
x	space coordinate

Greek symbols

α	thermal diffusivity
δ	freezing front coordinate
Δ	dimensionless freezing front coordinate
η	dimensionless coordinate
λ	freezing constant
θ	dimensionless temperature
ρ	density
σ	intermediate variable
τ	dimensionless time
ξ	intermediate variable
$\zeta, \zeta_1, \zeta_2, \zeta_3$	shape functions

Subscripts

app	approached value
exa	exact value
ref	reference
m	melting point

non-linear form of the thermal energy balance at the phase change front and the moving boundary nature of the solidification and melting problems, exact analytical solution are difficult to obtain except for problems which must be adaptable to similarity transformations. This restricts the problems to be solved in the infinite phase change material with constant thermophysical properties where the phase change takes place isothermally and is initiated by an imposed constant boundary temperature [1,2]. The exact solution recently obtained by a source and sink technique associated with Laplace transform approach concerns only problems with a moving heat front travelling at constant velocity in a fixed direction [3]. This situation is not relevant in nature and in practical applications. For example, in the heat storage or frozen food process, the boundary temperature is often time dependent.

With the severe limitation imposed by the exact solution, there have been many approximate approaches developed in the literature. The methods obtained, including numerical and analytical techniques, can be handled on two main approaches. One is the front-tacking method, where the