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Quasi-static thermal stresses in a thick circular plate

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Abstract

The present paper deals with the determination of a quasi-static thermal stresses in a thick circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature and the fixed circular edge thermally insulated. The results are obtained in series form in terms of Bessel's functions and they are illustrated numerically.
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1. Introduction

During the second half of the twentieth century, nonisothermal problems of the theory of elasticity became increasingly important. This is due to their wide application in diverse fields. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure.

Nowacki [1] has determined steady-state thermal stresses in circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge. Roy Choudhary [2,3] and Wankhede [4] determined Quasi-static thermal stresses in thin circular plate. Gogulwar and Deshmukh [5] determined thermal stresses in thin circular plate with heat sources. Also Tikhe and Deshmukh [6] studied transient thermoelastic deformation in a thin circular plate, where as Qian and Batra [7] studied transient thermoelastic deformation of thick functionally graded plate. Moreover, Sharma et al. [8] studied the behaviour of thermoelastic thick plate under lateral loads and obtained the results for radial and axial displacements and temperature change have been computed numerically and illustrated graphically for different theories of generalized thermoelasticity. Also Nasser [9,10] solved two-dimensional problem of thick plate with heat sources in generalized thermoelasticity. Recently Ruhi et al. [11] did thermoelastic

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analysis of thick walled finite length cylinders of functionally graded materials and obtained the results for stress, strain and displacement components through the thickness and along the length are presented due to uniform internal pressure and thermal loading.

This paper deals with the realistic problem of the quasi-static thermal stresses in a thick circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature and fixed circular edge thermally insulated. The results presented here will be more useful in engineering problem particularly in the determination of the state of strain in thick circular plate constituting foundations of containers for hot gases or liquids, in the foundations for furnaces, etc.

2. Formulation of the problem

Consider a thick circular plate of radius a and thickness h defined by $0 \leqslant r \leqslant a$, $-h/2 \leqslant z \leqslant h/2$. Let the plate be subjected to the arbitrary initial temperature over the upper surface ($z = h/2$) with the lower surface ($z = -h/2$) at zero temperature and the fixed circular edge thermally insulated. Under these more realistic prescribed conditions, the quasi-static thermal stresses are required to be determined.

The differential equation governing the displacement potential function $\phi(r, z, t)$ is given in [12] as

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = K\tau \tag{1}$$

$$\text{with} \quad \phi = 0 \text{ at } t = 0, \tag{2}$$

where K is the restraint coefficient and temperature change $\tau = T - T_{\mathrm{i}}$. T_{i} is initial temperature. Displacement function ϕ is known as Goodier's thermoelastic potential. The temperature of the plate at time t satisfies the heat conduction equation,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{3}$$

with the conditions

$$T = f(r) \quad \text{for } z = h/2,\ 0 \leqslant r \leqslant a, \quad \text{for all time } t, \tag{4}$$

$$T = 0 \quad \text{on } z = -h/2,\ 0 \leqslant r \leqslant a, \tag{5}$$

and

$$\frac{\partial T}{\partial r} = 0 \quad \text{at } r = a,\ -h/2 \leqslant z \leqslant h/2, \tag{6}$$

where k is the thermal diffusivity of the material of the plate.

The displacement function in the cylindrical coordinate system are represented by the Michell's function defined in [12] as

$$u_r = \frac{\partial\phi}{\partial r} - \frac{\partial^2 M}{\partial r\,\partial z}, \tag{7}$$

$$u_z = \frac{\partial\phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}. \tag{8}$$

The Michell's function M must satisfy

$$\nabla^2\nabla^2 M = 0, \tag{9}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{10}$$

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