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On an optimal shape design problem to control a thermoelastic deformation under a prescribed thermal treatment

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Abstract

A shape optimization problem concerned with thermal deformation of elastic bodies is considered. In this article, measure theory approach in function space is derived, resulting in an effective algorithm for the discretized optimization problem. First the problem is expressed as an optimal control problem governed by variational forms on a fixed domain. Then by using an embedding method, the class of admissible shapes is replaced by a class of positive Borel measures. The optimization problem in measure space is then approximated by a linear programming problem. The optimal measure representing optimal shape is approximated by the solution of this finite-dimensional linear programming problem. Numerical examples are also given.

Keywords: Linear programming; Optimal shape design; Measure theory; Thermoelasticity

1. Introduction

In many industrial applications, there is interest in control of the thermal deformation of an isotopic and homogeneous solid body subjected to a prescribed thermal treatment. Due to temperature changes, the body undergoes a thermoelastic deformation, that is, the induced thermal stress force the body to change its shape in time. As the final shape depends on the initial shape, one may be interested in finding the initial shape of the body such that its final shape after thermal treatment resembles a desired prescribed form as closely as possible. This problem may be expressed as an optimal shape design (OSD) problem.

A two-dimensional problem of this type was treated in [1] by transforming the system of equations onto a fixed domain. Another approach to solve this problem was given in [2], where the underlying domain was

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Nomenclature

t	time variable
T	final time
$x = (x_1, x_2)$	geometry coordinates before transformation
$\xi = (\xi_1, \xi_2)$	geometry coordinates after transformation
$s(x_1)$	unknown initial shape of the upper boundary
$\underline{s}$, $\bar{s}$	upper and lower bound for $s(x_1)$, respectively
$\overline{s'}$	upper bound for absolute value of $s'(x_1)$
Γ_1, Γ_2	fixed parts of the body shape
$\Gamma_3(s)$	moving part (upper boundary)of the body shape
$\Omega(s)$	the shape of the body before transformation
Ω	the shape of the body after transformation
$\hat{s}(x_1)$	desired shape for upper boundary at final time $t = T$
U^{ad}	the set of admissible shapes
$\theta(t, x)$	temperature before transformation
$\Theta(t, \xi)$	temperature after transformation
$\theta_0(x)$	initial temperature at $t = 0$
$\theta_s(x)$	temperature of the surrounding medium at the upper boundary
ρ	mass density
c	specific heat
κ	conductivity
α_s	convective heat transfer coefficient
τ_{ij}	components of stress tensors
ε_{ij}	components of linearized strain tensors
$u(t, x)$	displacement vector
$u(x)$	displacement vector at $t = T$
$u^1(t, x)$	horizontal component of displacement vector
$u^2(t, x)$	vertical component of displacement vector
E	Young's modulus
v_0	Poisson's ratio
α_{el}	coefficient of linear thermal expansion
F	body force
μ_0, λ_0	Lamé coefficients

discretized to handle the system of thermoelasticity by a finite element technique and to establish a descent method to determine the optimal shape.

In this article, we follow the procedure presented in [2] to convert the OSD problem to an optimal control problem. Then to each admissible control-state, a linear continuous functional is associated. Correspondence between continuous positive linear functionals and positive Borel measures leads to an optimization problem in measure space. The transformed problem in measure space is an appropriate formulation of the OSD problem since it is a linear programming (LP) problem in measure space. The solution of this LP problem is then approximated by the solution of a finite-dimensional LP problem which is attractive for consistent numerical computations. The sub-optimal shape will be found from the solution of the corresponding LP problem.

In comparison with other numerical methods for OSD problems, the proposed method in this article has some aspects. Gradient based methods [3,4] are restricted to differentiable cost functions but the proposed method may handles non-differentiable cost functions. Computation of the solution that uses finite element method (see [2,5]) is a time consuming task but presented method in this article is not iterative and therefore, it is self-starting and dose not need any initial solution. Methods described in [2,4] that use control-state