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Applied Mathematical Modelling 31 (2007) 393–403
APPLIED MATHEMATICAL MODELLING
www.elsevier.com/locate/apm

On an EPQ model for deteriorating items under permissible delay in payments

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Received 1 October 2004; received in revised form 1 March 2005; accepted 1 November 2005
Available online 4 January 2006

Abstract

This paper derives a production model for the lot-size inventory system with finite production rate, taking into consideration the effect of decay and the condition of permissible delay in payments, in which the restrictive assumption of a permissible delay is relaxed to that at the end of the credit period, the retailer will make a partial payment on total purchasing cost to the supplier and pay off the remaining balance by loan from the bank. At first, this paper shows that there exists a unique optimal cycle time to minimize the total variable cost per unit time. Then, a theorem is developed to determine the optimal ordering policies and bounds for the optimal cycle time are provided to develop an algorithm. Numerical examples reveal that our optimization procedure is very accurate and rapid. Finally, it is shown that the model developed by Huang [1] can be treated as a special case of this paper.

Keywords: EPQ; EOQ; Trade credit; Delay payment; Deteriorating items; Inventory

1. Introduction

The classical inventory model considers the ideal case that the value of inventory items are unaffected by time and replenishment is done instantaneously. In real life cases, however, the ideal case is not quit applicable. Inventories are often replenished periodically at certain production rate which is seldom infinite. Even for purchased items, when supply arrives at the warehouse, it may take days for receiving department to completely transfer the supply into storage room. Goods deteriorate and their value reduces with time. Electronic products may become obsolete as technology changes; fashion tends to depreciate the value of clothing over time; batteries die out as they age. The effect of time is even more critical for perishable goods such as foodstuff and cigarettes. The effect of these two situations is that the classical inventory model has to be readjusted.

More recently, the supplier will offer the retailer a trade credit period in a competitive market environment, in paying for the amount of purchasing cost. Usually, there is no charge if the outstanding amount is settled

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0307-904X/$ - see front matter
doi:10.1016/j.apm.2005.11.016

within the permitted fixed settlement period. Beyond this period, interest is changed. As a matter of fact, the permissible delay in payments produces a benefit to the supplier is that the policy should attract new customers who consider it to be a type of price reduction and increase sales. In recent research, the extensive use of trade credit as an alternative has been addressed by Goyal [2] who developed an economic order quantity (EOQ) model under the conditions of permissible delay in payments. Chung [3] then developed an alternative approach to the problem. Chand and Ward [4] analyzed Goyal's problem under assumptions of the classical economic order quantity model, obtaining different results. Next, Aggarwal and Jaggi [5] extended Goyal's model to allow for deteriorating items. Jamal et al. [6] extended Aggarwal and Jaggi's [5] model to shortages. There were several interesting and relevant papers related to the delay of payments such as Chu et al. [7], Chung [8], Hwang and Shinn [9], Jamal et al. [10], Sarker et al. [11,12], Shah [13], Shinn [14], Khouja and Mehrez [15] and their references. However, these studies were developed under the assumption that the items obtained from an outside supplier and the entire lot size is delivered at the same time. In fact, when an item can be produced in-house, the replenishment rate is also the production rate, and is hence finite. Hence, we amen Goyal's model by considering the replenishment rate is finite, the difference between purchasing price and selling cost and taking into consideration the effect of time or decay.

Specifically, the restrictive assumption of a permissible delay is relaxed to that at the end of the credit period, the retailer will make a partial payment on total purchasing cost and pay off the remaining balance by loan from the bank. Consequently, the main purpose of this paper is to show that there exists a unique optimal cycle time to minimize the total variable cost per unit time. Next, a simple theorem to determine the optimal cycle time will be developed. Bounds for the optimal cycle time are provided to develop the algorithm and numerical examples are also presented to illustrate the results of the proposed model. Finally, it is shown that the model developed by Huang [1] can be treated as a special case.

2. Mathematical model and analysis

The mathematical model of the inventory system considered in this paper is basically an extension of the work of Huang [1] and is developed on the basis of the following assumptions and notations:

Notation

D	annual demand rate
P	annual replenishment rate, $P > D$
A	cost of placing one order
ρ	$1 - \frac{D}{P} > 0$
c	unit purchasing price per item
s	unit selling price per item of good quality
h	unit stock holding cost per item per year excluding interest charges
I_e	interest which can be earned per \$ per year
I_p	interest charges per \$ investment in inventory per year
$I_1(t)$	the inventory level that changes with time t during production period
$I_2(t)$	the inventory level that changes with time t during non-production period
M	the trade credit period
T	the cycle time
T^*	the optimal cycle time
θ	deterioration rate of finished item

Assumptions:

(1) Demand rate is known and constant.
(2) Replenishment rate, P, is known and constant.
(3) Shortages are not allowed.
(4) The constant fraction θ of on hand inventory gets deteriorated per time unit.
(5) Time period is infinite.