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Characteristic development of hyperbolic two-dimensional two-fluid model for gas–liquid flows with surface tension

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Abstract

A new hyperbolic, two-dimensional two-fluid model is developed to properly solve two-phase gas–liquid flows. Adopting the interfacial pressure jump terms in the momentum equations, the numerical stability is confirmed owing to the improvement in the mathematical property of the equation system. The derivation of the interfacial pressure jump terms is based on the infinitesimal surface-tension effect incorporated in the pressure difference at the gas–liquid interface. Through the characteristic analysis on the equation system, the eight eigenvalues are obtained analytically and they are proved real values representing phasic convective velocities and phasic sound speeds. Furthermore, the characteristic sound speeds are comparable with the earlier experimental data in excellent agreements. In addition, the eigenvectors are obtained analytically and they are shown to be linearly independent. Consequently, the governing equation system is mathematically hyperbolic with reasonable characteristic speeds by which the upwind numerical method avails. Advantage and possibility of the present model are discussed in some detail.

Keywords: Two-phase flow; Two-fluid model; Interfacial pressure jump; Surface tension; Sound speed; Eigenvalues

1. Introduction

Conventional equation systems of one-dimensional two-phase flow with the same phasic pressure having complex eigenvalues inherently become ill-posed initial value problem as indicated by Lyczkowski et al. [1], Ramshaw and Trapp [2], and Stewart [3]. The complex eigenvalues of the equation system induce the unbounded exponential growth of a small-amplitude disturbance. This non-physical instability has been damped out by artificial numerical techniques or additional momentum sources like artificial viscosity or virtual mass terms. During the past decades, a lot of contribution has been focused not only on the governing equations rendering real eigenvalues but also on numerical methods calculating stable computational results. Especially, introducing physical terms was known to improve mathematical properties of the governing equations by representing the real eigenvalues, reported by Travis [4], Stunmiller [5], and so on.

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Without considering the hyperbolicity of equation system, Markatos and Kirkcaldy [6] developed a well-defined continuum mixture approach for three-dimensional gas–solid two-phase flow on the basis that mass, momentum and energy balanced over control volumes occupied by space-sharing interspersed continua. The formulations of pressure terms represented a single pressure for gas–solid two-phase system and an additional intergranular-force term in one of the equations. In the equation system, the terms proportional to pressure whenever void fraction is non-uniform were omitted. From the well-defined model and the numerical method called IPSA (inter-phase slip algorithm) with staggered grid, they showed stable fast-converging solutions treating two-phase unsteady reactive flows.

In addition, Markatos [7] also presented a stable, fast-converging, fully implicit numerical method, first proposed by Spalding, which was used to solve coupled, non-linear partial differential equations. Some discussions were given of the sensitivity of his predictions to the assumed constitutive relations for interface heat/momentum transfer and the hyperbolicity of the equations in spite of their unique solutions.

However, for the gas–liquid two-phase flow, such terms as void fraction gradients actually contribute momentum sources when considering the pressure difference of each fluid exists at the interfacial surface. Furthermore, if the equation system could be changed hyperbolic, an upwind method depending on the signal propagation can be used to solve the equation system treating the abrupt change of fluid properties instead of using staggered grid: Based on the condition that the eigenvalues of the equation system are characteristic speed of fluids such as convective speed, gas and liquid sound speeds, the single phase compressible flow has been able to adopt the higher-order upwind method.

Fortunately, Lee et al. [8] suggested a new hyperbolic two-fluid model for the area-averaged simulation of one-dimensional two-phase flow and it has been improved and applied to analyze various two-phase flow problems. Therefore, in order to simulate two-dimensional two-phase flow problems with abrupt change of fluid properties effectively, it is possible to develop a new hyperbolic two-dimensional model based on the above one-dimensional equations and to adopt an upwind numerical method like Roe's approximate Riemann solver or flux vector splitting (FVS) method.

The upwind methods applied to the hyperbolic two-fluid models can be found in the following references. For examples, Toumi [9] used the Roe's scheme to calculate one-dimensional two-fluid equations including virtual mass terms. Stadke et al. [10] also applied the flux vector splitting method to the two-dimensional two-fluid model. However, the above models used in Refs. [9,10] depended on the experimental factors like virtual mass coefficients and an artificial viscosity.

For this reason, we suggest a new two-dimensional two-fluid model from the one-dimensional model, which has been developed by Chung et al. [11]. Because the resulted two-dimensional model becomes mathematically hyperbolic depending only on the fluid properties as will be shown in Section 3, the upwind method can be adopted to solve this equation system efficiently.

In Section 2, we will suggest the two-dimensional two-fluid model considering the effect of mass, momentum, and energy interactions to simulate more realistic two-phase flow than the conventional model did. Before the generalization of this two-dimensional model, we assumed here that the flow pattern is focused on the homogeneous bubbly flow with variation of void fraction in the x/y-directions. In Section 3, we will analyze that the mathematical property of the two-dimensional two-fluid model based on the characteristic analysis. After that, we will validate the physical properties of the system eigenvalues by comparing the experimental data and discuss on the comparison of the results in some detail.

2. Two-dimensional two-fluid model

2.1. Governing equations

Treating the two-phase water–air/water–vapor flow, we assumed that the flow pattern is homogeneous bubbly flow having evaporation and condensation for simplicity of mathematical modeling. Therefore, the governing equations of the two-dimensional two-phase flow are derived from the mass, momentum, and entropy conservation laws as follows.