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Nonlinear analysis of thin rectangular plates on Winkler–Pasternak elastic foundations by DSC–HDQ methods

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Abstract

This article introduces a coupled methodology for the numerical solution of geometrically nonlinear static and dynamic problem of thin rectangular plates resting on elastic foundation. Winkler–Pasternak two-parameter foundation model is considered. Dynamic analogues Von Karman equations are used. The governing nonlinear partial differential equations of the plate are discretized in space and time domains using the discrete singular convolution (DSC) and harmonic differential quadrature (HDQ) methods, respectively. Two different realizations of singular kernels such as the regularized Shannon's kernel (RSK) and Lagrange delta (LD) kernel are selected as singular convolution to illustrate the present DSC algorithm. The analysis provides for both clamped and simply supported plates with immovable inplane boundary conditions at the edges. Various types of dynamic loading, namely a step function, a sinusoidal pulse, an *N*-wave pulse, and a triangular load are investigated and the results are presented graphically. The effects of Winkler and Pasternak foundation parameters, influence of mass of foundation on the response have been investigated. In addition, the influence of damping on the dynamic analysis has been studied. The accuracy of the proposed DSC–HDQ coupled methodology is demonstrated by the numerical examples.

Keywords: Nonlinear analysis; Discrete singular convolution; Differential quadrature; Rectangular plates; Winkler foundation; Pasternak foundation; Dynamic analysis

1. Introduction

Plates and shells with or without on elastic foundation have found wide applications as component in practical engineering structures such as aerospace, biomechanics, petrochemical, marine industry, civil and mechanical engineering applications. There are many situations such as seismic tests, nuclear explosions, earthquakes, etc. in which these structures are subjected to transient loads and large amplitudes of motion may occur. Therefore, dynamic analysis of such components is important for the safety and stability of the structures.

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Nonlinear static and dynamic analyses of plates of various shapes have been carried out by various researchers [1–5]. More detailed information can be found in a recent review paper by Sathyamorth [6]. Nath and Kumar [7] used Chebyshev polynomials for nonlinear analysis of plates. Nath et al. [8] presented the finite differences methods for spatial discretization and Houbolt's time marching discretization to study the dynamic analysis of rectangular plates resting on elastic foundation. Dumir [9] and Dumir and Bhaskar [10] have investigated nonlinear static and dynamic analysis of rectangular plates on elastic foundation employing the orthogonal point collocation method. A few studies concerning to static and dynamic analysis of rectangular plates resting on elastic foundation have been carried out, namely by Cheung and Zienkiewicz [11], Yang [12], Liew et al. [13], Liu [14].

In this paper, an approximate numerical solution of the Von Karman–Donnel type governing equations are proposed for the geometrically nonlinear analysis of thin rectangular plates resting on Winkler–Pasternak elastic foundations. For this purpose, the discrete singular convolution (DSC) and harmonic differential quadrature (HDQ) methods had been used for spatial and temporal discretization of governing differential equations of problem. To the authors' knowledge, it is the first time the DSC method has been successfully applied to rectangular plates resting on two parameter elastic foundation problem for the geometrically nonlinear dynamic analysis. The accuracy and efficiency of DSC–HDQ are demonstrated throughout the numerical examples. This article is organized into six sections. A brief review of the DSC method is given in Section 2. HDQ formulations of temporal discretization of the governing equations are given in Section 3. In Section 4, governing equations and solution procedures are presented. The numerical results of DSC–HDQ methods are presented and discussed in Section 5. Finally, a conclusion is given in Section 6.

2. Discrete singular convolution (DSC)

The discrete singular convolution (DSC) algorithm was originally introduced by Wei [15] in 1999 as a simple and highly efficient numerical technique. As stated by Wei [16] singular convolutions (SC) are a special class of mathematical transformations, which appear in many science and engineering problems, such as the Hilbert, Abel and Radon transforms.

In a general definition, numerical solutions of differential equations are formulated by some singular kernels. The mathematical foundation of the DSC algorithm is the theory of distributions [17] and wavelet analysis. Theory of wavelets developed in recent years has great impact in telecommunication and electronic engineering and has found their application in a variety of other science and engineering disciplines. Mathematically, wavelets are functions generated from a single function by dilation and translation. In fact, the theory of wavelets and frames, a new mathematical branch developed in recent years.

In this paper, details of DSC method are not given; interested readers may refer to the works of Wei [19] and Wei et al. [18,20] who originated the method and recent survey paper of the present investigators [21,22].

Wei and his co-workers first applied the DSC algorithm to solve solid and fluid mechanics problem [18,23–25]. Wan et al. [26] applied the DSC method for the numerical solution of unsteady incompressible flows. The high frequency vibration analysis of plates using DSC algorithm is given by Zhao et al. [27]. Zhao and Wei [28] adopted the DSC in the vibration analysis of rectangular plates with nonuniform boundary conditions. Wei investigated numerical solution of sine-Gordon equation by the algorithm of DSC [29]. Recently, a good comparative accuracy of DSC and generalized differential quadrature methods for vibration analysis of rectangular plates is presented by Ng et al. [30]. Most recently, Hou et al. [31] and Lim et al. [32] presented the DSC-Ritz method for the free vibration analysis of Mindlin plates and thick shallow shells. These studies indicates that the DSC algorithm work very well for the vibration analysis of plates, especially for high-frequency analysis of rectangular plates. Furthermore, it is also concluded that the DSC algorithm has global methods' accuracy and local methods' flexibility for solving differential equations in applied mechanics.

Discrete singular convolution (DSC) is an effective approach for the numerical realization of singular convolutions, which occur commonly in science and engineering. Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t - x)\eta(x)\,\mathrm{d}x, \tag{1}$$