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# A wavelet-based stochastic finite element method of thin plate bending

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## Abstract

A wavelet-based stochastic finite element method is presented for the bending analysis of thin plates. The wavelet scaling functions of spline wavelets are selected to construct the displacement interpolation functions of a rectangular thin plate element and the displacement shape functions are expressed by the spline wavelets. A new wavelet-based finite element formulation of thin plate bending is developed by using the virtual work principle. A wavelet-based stochastic finite element method that combines the proposed wavelet-based finite element method with Monte Carlo method is further formulated. With the aid of the wavelet-based stochastic finite element method, the present paper can deal with the problem of thin plate response variability resulting from the spatial variability of the material properties when it is subjected to static loads of uncertain nature. Numerical examples of thin plate bending have demonstrated that the proposed wavelet-based stochastic finite element method can achieve a high numerical accuracy and converges fast.


*Keywords:* Wavelet function; Wavelet finite element; Spline; Thin plate; Bending

## 1. Introduction

The reliability of many engineering structures in the presence of uncertainty has been a crucial issue in their analysis and design. Primary systems related to civil engineering structures may be quite sensitive to small imperfections of pertinent design variables. These variables may include the quantities such as modulus of elasticity, Poisson's ratio, shear strength, applied loads, and a variety of other physical and mathematical parameters [1–4]. Several of these variables are inherently random and can be maximal appropriately modeled as random processes. Clearly, the complexity of civil engineering structures requires the use of versatile numerical algorithms, such as the finite element method, to obtain the mathematical approximation to their physical

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behavior. A challenging task, however, is to accurately account for the randomness in a given problem while using some proven numerical algorithms.

Finite element method (FEM) based on the energy variational principle and discrete interpolation has been developed since the middle of twenty century, which makes a full use of advantages of conventional energy method and finite difference method. FEM is now a powerful tool in structural analysis and design [5–11]. In the development of FEM, the energy variational principle provides a theoretical foundation to develop varieties of finite element formulations. The most developed one, i.e. the finite element method provides the necessary modeling flexibility. Standard deterministic form of the finite element tool modified to the stochastic finite element method has been developed to analyze system stochasticity problems. Extensive reviews regarding this can be found [12–14].

Wavelet theory is a mathematical tool that was developed in recent decades. As a new branch of mathematics, it has gained more and more attentions in engineering fields, such as signal processing, image processing, pattern and phonetic recognition, quantum physics, earthquake reconnaissance, fluid mechanics, diagnosis and monitoring of machinery defect, etc. The wavelet analysis based on the wavelet transform has the extraordinary characteristics that combine the advantages of functional analysis, Fourier transform, spline analysis, harmonic analysis and numerical analysis [15–18]. It is convinced that the wavelet based methods are powerful in analyzing the field problems with changes in gradients and singularities due to the excellent multi-resolution properties of wavelet functions.

Several investigators have tried to integrate the wavelet analysis with the finite element concept to solve various filed problems. The wavelet-decomposed method was proposed to solve the Navier–Stokes equations [19]. Some investigators combined the Garlerkin method and wavelet analysis, which was called the wavelet Garlerkin method. This method had been applied to solve the Dirichlet problem [20]. Daubechies wavelet has been used to construct one-dimensional Daubechies wavelet beam element [21]. The numerical algorithm of Daubechies wavelet derivative with a higher order was presented, which was used to solve the differential equations of beam and plate [22,23]. However, the current wavelet-based finite elements were always constructed in the wavelet space where the wavelet coefficients were used as the parameters to be determined [24,25]. In this way, when a complex structural problem is analyzed, the interfaces between different elements and boundary conditions cannot be easily treated as the conventional displacement-based finite element methods do. This shortcoming limits a wide application of wavelet-based finite element methods. Recently, authors [26] proposed a multivariable wavelet-based finite element method with the help of the Hellinger–Reissner generalized variational principle. The formulation has been successfully applied to solve the thick plate problems.

The bending of a thin plate is a two-dimensional problem. Various formulations of robust thin plate bending elements have been proposed [27–31]. In this paper, a new wavelet-based stochastic finite element method is presented by using the spline wavelets. To overcome above shortcoming of wavelet-based elements and to have a wide range of applicability, the present spline wavelet finite element formulation of thin plate bending is constructed as the same way of conventional displacement-based finite element method. The spline wavelet functions are used as the displacement interpolation functions of thin plates and corresponding shape functions are derived. Combining the Monte Carlo method, a stochastic wavelet-based finite element formulation is to deal with the problem of the thin plate response variability resulting from the spatial variability of the material properties when it is subjected to static loads of uncertain nature. Numerical examples illustrate that current wavelet-based stochastic finite element method has a high analytical accuracy.

## 2. Spline wavelets and scaling functions

Wavelet analysis is fundamentally a different approach. The wavelet theory is based on the idea that any signal can be broken down a series of local basis functions called ‘‘wavelet’’. Any particular local feature of a signal can be analyzed according to the scale and translation characteristics of wavelets. Due to the compact support and orthogonality, the spline wavelet can describe the details of the field problem conveniently and accurately. More importance is that the spline wavelet has the explicit expressions which facilitate not only the theoretical formulation, but also numerical implementations with computers. To construct spline wavelet based finite elements, the properties of spline wavelets is briefly discussed as follows. More detailed description can be found in Ref. [17].