Available online at www.sciencedirect.com
SCIENCE DIRECT®
Applied Mathematical Modelling 30 (2006) 177–199
APPLIED MATHEMATICAL MODELLING
www.elsevier.com/locate/apm

Finite element analysis of laminar and turbulent flows using LES and subgrid-scale models

T.L. Popiolek [a,*], A.M. Awruch [b], P.R.F. Teixeira [a]

[a] *Department of Mathematics, Federal University Foundation of Rio Grande, Av. Itália km 8, 96200-000 Rio Grande, RS, Brazil*

[b] *Graduate Program in Civil Engineering, Federal University of Rio Grande do Sul, Av. Osvaldo Aranha 99 -3° andar, 90035-190, Porto Alegre, RS, Brazil*

Received 1 November 2003; accepted 18 March 2005
Available online 17 May 2005

Abstract

Numerical simulations of laminar and turbulent flows in a lid driven cavity and over a backward-facing step are presented in this work. The main objectives of this research are to know more about the structure of turbulent flows, to identify their three-dimensional characteristic and to study physical effects due to heat transfer. The filtered Navier–Stokes equations are used to simulate large scales, however they are supplemented by subgrid-scale (SGS) models to simulate the energy transfer from large scales toward subgrid-scales, where this energy will be dissipated by molecular viscosity. Two SGS models are applied: the classical Smagorinsky's model and the Dynamic model for large eddy simulation (LES). Both models are implemented in a three-dimensional finite element code using linear tetrahedral elements. Qualitative and quantitative aspects of two and three-dimensional flows in a lid-driven cavity and over a backward-facing step, using LES, are analyzed comparing numerical and experimental results obtained by other authors.

Keywords: Laminar and turbulent flows; Large eddy simulation; Finite elements; Subgrid-scale model

* Corresponding author. Fax: +55 5323 15382.
E-mail address: dmttales@furg.br (T.L. Popiolek).

0307-904X/$ - see front matter
doi:10.1016/j.apm.2005.03.019

1. Introduction

Turbulent flows are of great practical interest in several engineering fields and may be defined as a three-dimensional flow with highly disordered, intermittent and rotational fluid motion and with diffusive and dissipative characteristics. Although formulation of mathematical models to simulate numerically such complex flows is a challenging task, many researches have been developed and some reliable results have been obtained.

Flows with high Reynolds numbers, where the influence of the different turbulence scales must be taken into account, cannot be solved by direct numerical simulation (DNS) due to the large amount of data and unknowns involved in the computational solution (and the corresponding requirements in terms of CPU time and computer memory).

Turbulent flows may be simulated using the Reynolds Averaged Navier–Stokes (RANS) equations. This approach is based in the separation of the instantaneous value of a specific flow variable in its mean value and fluctuations with respect to this mean value. The well-known Reynolds stress components are originated substituting mean values and fluctuations of the variables in the conservation equations. Details about this subject can be found in traditional texts such as Hinze [1], Schlichting [2] and Tennekes and Lumley [3], among others.

The RANS equations have more unknowns than equations, and for this reason it is necessary to use closure models to define the Reynolds stress components. Several models have been employed by different authors in the last three decades, and most of these models are described by the state of the art reviews presented by Launder and Spalding [4,5], Rodi [6] and Markatos [7], among others. In recent years several authors have implemented different modifications to the original equations in order to get some improvements of the numerical models behaviour.

Alternatively, large eddy simulation (LES) may be used to analyze turbulent flows. This methodology was initially proposed by Smagorinsky [8], and it consists in the separation of the large eddies and subgrid-scales using a grid filter. Large eddies are associated to the low flow frequencies and they are originated by the domain geometry and the boundaries. Subgrid-scales (SGS) are associated to high frequencies and they have an isotropic and homogeneous behaviour, maintaining their independence with respect to the main stream. As in RANS equations, in LES is also necessary to use closure models, and due to the characteristics of the SGS (homogeneity, isotropy and no significant variations for different flows), they are more appropriated to be represented by mathematical models. Then, in LES the Large Eddies are simulated directly, whereas SGS are simulated using closure models. Although RANS equations and LES seem to be similar, in the first one the closure models simulate the momentum and energy transfer from the mean flow to the fluctuating part, while in LES the closure models simulate the momentum and energy transfer from the large eddies to the small turbulence scales (or subgrid-scales). Comparisons, advances and trends of turbulence models applied to bluff bodies were presented by Ferzinger [9], Leschziner [10] and Murakami [11].

In this work, studies to simulate turbulent flows using LES, with the classical Smagorinsky's model and the dynamic subgrid-scale model are presented. The three-dimensional flow in a lid-driven cavity is simulated, and statistical studies with respect to the velocity mean value, turbulence intensity and Reynolds stresses are performed. Two and three-dimensional flows in a backward-facing step are also analyzed in order to verify the behaviour of the two models comparing results of this work with those obtained numerically and experimentally by other authors.