Applied Mathematics Letters 64 (2017) 1–7

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Boundedness in a parabolic–elliptic chemotaxis-growth system under a critical parameter condition

Bingran Hu[a], Youshan Tao[b,*]

[a] *School of Information & Technology, Dong Hua University, Shanghai 200051, PR China*
[b] *Department of Applied Mathematics, Dong Hua University, Shanghai 200051, PR China*

ARTICLE INFO

Article history:
Received 10 July 2016
Received in revised form 7 August 2016
Accepted 7 August 2016
Available online 16 August 2016

Keywords:
Boundedness
Chemotaxis-growth system
Critical parameter condition

ABSTRACT

We consider the parabolic–elliptic chemotaxis-growth system

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u^m \nabla v) + \mu u(1-u^\alpha), & x \in \Omega,\ t > 0, \\ -\Delta v + v = u^\gamma, & x \in \Omega,\ t > 0, \end{cases}$$

under no-flux boundary conditions in a smoothly bounded domain $\Omega \subset \mathbb{R}^N$, $N \geq 1$, where χ, μ, m, α and γ are prescribed positive parameters fulfilling $m \geq 1$ and $\gamma \geq 1$.

Recently, it has been proved in Galakhov et al. (2016) that if either $\alpha > m+\gamma-1$ or $\alpha = m+\gamma-1$ and $\mu > \frac{N\alpha-2}{2(m-1)+N\alpha}\chi$, for any given $u_0 \in W^{1,\infty}(\Omega)$ this system possesses a global and bounded classical solution. The present work further shows that the same conclusion still holds for the critical case $\alpha = m+\gamma-1$ and $\mu = \frac{N\alpha-2}{2(m-1)+N\alpha}\chi$.

1. Introduction

Chemotaxis, known as a biased movement of cells along the concentration gradient of a cue, plays a central role in colorful biological processes such as pattern formation, bacteria aggregation, angiogenesis and cancer invasion. The celebrated chemotaxis model was initially proposed by Keller and Segel in 1970 [1], and it has been comprehensively investigated in past four decades (see [2], for instance). Motivated by various biological phenomena, numerous variants of the Keller–Segel model have been developed [2]. Among them, some recent works qualitatively study the effects of interplay between self-diffusion and cross-diffusion [3,4], between self-diffusion and logistic damping [5], or between nonlinear signal production and logistic growth [6]

* Corresponding author.
E-mail addresses: hubingran@qq.com (B. Hu), taoys@dhu.edu.cn (Y. Tao).

http://dx.doi.org/10.1016/j.aml.2016.08.003
0893-9659/

on the properties of solutions to the corresponding modified models. In order to address the dependence of dynamical behaviors of solutions on the interactions between nonlinear cross-diffusion, generalized logistic source and superlinear signal production, Galakhov et al. [7] recently considered the initial–boundary value problem

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u^m\nabla v) + \mu u(1-u^\alpha), & x\in\Omega,\ t>0,\\ -\Delta v + v = u^\gamma, & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

in a smoothly bounded domain $\Omega\subset\mathbb{R}^N$, $N\ge 1$, where χ,μ,m,α and γ are given positive parameters.

For the prototype case $m=\alpha=\gamma=1$, it is shown that if $\mu > \frac{(N-2)_+}{N}\chi$, then for any given initial data u_0 with suitable regularity the system admits a global and bounded solution [8]. As for the critical case $\mu=\frac{N-2}{N}$, $N\ge 3$, the global existence of classical solution was also asserted in [9]; however we should point out that the possibility of infinite-time blow-up of solutions for this critical case is not ruled out in [9]. For the general case $m\ge 1$ and $\gamma\ge 1$, it has been shown that if either $\alpha > m+\gamma-1$ or $\alpha = m+\gamma-1$ and $\mu > \frac{N\alpha-2}{2(m-1)+N\alpha}\chi$, for any given $u_0\in W^{1,\infty}(\Omega)$ the system (1.1) possesses a global and bounded classical solution [7].

Inspired by the above-mentioned latter two recent works [7,9], the present work focuses on the analysis of (1.1) for the critical case $\alpha = m+\gamma-1$ and $\mu = \frac{N\alpha-2}{2(m-1)+N\alpha}\chi$ with $N\ge 3$. Our result not only claims the global existence of solutions but also excludes the possibility of infinite-time explosion. More precisely, we have:

Theorem 1.1. *Let $\Omega\subset\mathbb{R}^N$, $\mathbb{N}\ge 3$, be a bounded domain with smooth boundary, suppose that χ,μ,m,α and γ are given positive parameters fulfilling*

$$m\ge 1 \quad \text{and} \quad \gamma\ge 1, \tag{1.2}$$

and assume that

$$\alpha = m+\gamma-1 \quad \text{and} \quad \mu = \frac{N\alpha-2}{2(m-1)+N\alpha}\chi. \tag{1.3}$$

Then for any given nonnegative $u_0\in W^{1,\infty}(\Omega)$ the problem (1.1) possesses a global classical solution (u,v) which is bounded in $\Omega\times(0,\infty)$ in the sense that there exists $C>0$ satisfying

$$\|u(\cdot,t)\|_{L^\infty(\Omega)} + \|v(\cdot,t)\|_{L^\infty(\Omega)} \le C \quad \text{for all } t>0. \tag{1.4}$$

Finally, we make a comparison between our method and the argument in [7]. In order to establish a bound for $\int_\Omega u^p(\cdot,t)$, $t\in(0,T_{\max})$, the standard entropy-like inequality is derived (see Lemma 2.3). The term $\int_\Omega u^{p+\alpha}$ on the rightmost of (2.3) is *controllable in the sub-critical case* (see Lemma 2.3 in [7] for details); however, it is *uncontrollable in the critical case*. Making full use of the negativity of the coefficient $A(p)$ before $\int_\Omega u^{p+\alpha}$ whenever p is smaller than some p_1 and employing the smallness of $A(p)$ when $p>p_1$ but close to p_1, we derive a bound for $\int_\Omega u^{p_1+\delta}(\cdot,t)$, $t\in(0,T_{\max})$, with some small $\delta>0$ via a bootstrap argument (see Lemmata 2.4–2.6). Relying on this, we further invoke the bootstrap argument once again to raise the L^p-integrality of u (see Lemma 2.7) and thereby complete the proof of our main claim.

2. Proof of the main result

We begin with the local existence and the extensibility criterion (cf. [10,8], for instance).