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Homoclinic solutions in non-periodic discrete ϕ-Laplacian equations with mixed nonlinearities

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ABSTRACT

By using critical point theory, we obtain some sufficient conditions on the existence of homoclinic solutions of a class of non-periodic discrete ϕ-Laplacian equations. In our paper, the nonlinearities can be mixed super p-linear with asymptotically p-linear at ∞ for $p \geq 1$, and be mixed superlinear with asymptotically linear at 0. To the best of our knowledge, there is no such result for the existence of homoclinic solutions in non-periodic discrete ϕ-Laplacian equations before. Some results in the literature are improved.

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1. Introduction

In the past years, the discrete nonlinear Schrödinger (DNLS) equation, which is a nonlinear lattice system that appears in many areas of physics, received great attention. Discrete solitons which exist in the DNLS systems also arouse much interest, for example, photorefractive media [1], biomolecular chains [2] and Bose–Einstein condensates [3]. The experimental observations of discrete solitons in nonlinear lattice systems have been reported in [4]. Many authors have studied the existence of discrete solitons of the DNLS equations. To mention a few, see [5–8]. And many methods are used, for example, the principle of anticontinuity [5], variational methods [6], centre manifold reduction [7] and the Nehari manifold approach [8].

Consider the discrete solitons of the following DNLS equation with attractive self-interaction:

$$i\dot{\psi}_n = -\Delta\psi_n + v_n\psi_n - \gamma_n g_n(\psi_n), \quad n \in \mathbb{Z}, \tag{1.1}$$

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where $\Delta\psi_n = \psi_{n+1} + \psi_{n-1} - 2\psi_n$ is the discrete Laplacian in one spatial dimension, the given sequence $\{v_n\}_{n\in\mathbb{Z}}$ is assumed to be positive, $\gamma = \{\gamma_n\}_{n\in\mathbb{Z}}$ is a positive real valued sequence, and $g_n(u)$ is a nonlinear function in u for each $n \in \mathbb{Z}$. The function g_n for each $n \in \mathbb{Z}$ is said to be super $(p-1)$-linear or asymptotically $(p-1)$-linear at ∞ for some $p \geq 2$ if

$$\lim_{|u|\to\infty} \frac{g_n(u)}{|u|^{p-2}u} = \infty \quad \text{or} \quad \lim_{|u|\to\infty} \frac{g_n(u)}{|u|^{p-2}u} = a_n < \infty,$$

respectively. Typical representatives of nonlinearity are

$$g_n(u) = c_n|u|^{p-2}u, \quad c_n \neq 0,\ p > 2, \tag{1.2}$$

and

$$g_n(u) = l_n \frac{|u|^{q+p-2}u}{1+|u|^q}, \quad l_n \neq 0,\ q > 0,\ p \geq 2.$$

Considering (1.1), assume the nonlinearity is gauge invariant, i.e.,

$$g_n(e^{i\theta}u) = e^{i\theta}g_n(u), \quad \theta \in \mathbb{R},$$

and, in addition, $g_n(u) \geq 0$ for $u \geq 0$.

By the discrete soliton Ansatz, we change the discrete solitons for (1.1) to a stationary problem:

$$\psi_n = u_n e^{-i\omega t} \quad \text{and} \quad \lim_{|n|\to\infty} \psi_n = 0,$$

where $\{u_n\}_{n\in\mathbb{Z}}$ is a real valued sequence and $\omega \in \mathbb{R}$ is the temporal frequency. Then (1.1) becomes

$$-\Delta u_n + v_n u_n - \omega u_n = \gamma_n g_n(u_n), \quad n \in \mathbb{Z}, \tag{1.3}$$

and

$$\lim_{|n|\to\infty} u_n = 0 \tag{1.4}$$

holds.

Assume that $g_n(0) = 0$ for each $n \in \mathbb{Z}$, then $\{u_n\}_{n\in\mathbb{Z}} = \{0\}$ is a solution of (1.3), which is called the trivial solution. As usual, we say that a solution $u = \{u_n\}_{n\in\mathbb{Z}}$ of (1.3) is homoclinic (to 0) if (1.4) holds. In addition, if $\{u_n\}_{n\in\mathbb{Z}} \neq \{0\}$, then u is called a nontrivial homoclinic solution. Clearly, discrete solitons of (1.1) correspond to the homoclinic solutions of (1.3). We point out that the existence of homoclinic solutions of (1.3) has been studied in [9,10].

Actually, in this paper, we will study the existence of the nontrivial homoclinic solutions of the following nonlinear discrete ϕ-Laplacian equation:

$$-\triangle(\phi(\triangle u_{n-1})) + \omega_n u_n = \gamma_n g_n(u_n), \quad n \in \mathbb{Z}, \tag{1.5}$$

where $\triangle u_n = u_{n+1} - u_n$, $\phi(u)$ is continuous in u with $\phi(0) = 0$, and $\Omega = \{\omega_n\}_{n\in\mathbb{Z}}$ is a positive real valued sequence. Obviously, (1.3) is a special form of (1.5) with $\phi(u) = u$ and $\omega_n = v_n - \omega$ for each $n \in \mathbb{Z}$.

The aims of this paper read as follows. First, (1.3) was considered by us in [10] when g_n for each $n \in \mathbb{Z}$ is superlinear at both ∞ and 0, which plays an important role in the existence of homoclinic solutions of (1.3). In our previous work [11], we considered homoclinic solutions in periodic difference equations with mixed nonlinearities. In this paper, the nonlinearities can be mixed super p-linear with asymptotically p-linear at ∞ for $p \geq 1$, and be mixed superlinear with asymptotically linear at 0. To the best of our knowledge, there is no such result for (1.5) with mixed nonlinearities. In addition, the nonlinearity like (1.2) satisfies $\frac{g_n(u)}{|u|}$ being nondecreasing with respect to $|u|$ for each $n \in \mathbb{Z}$. In this paper, the monotonicity of $\frac{g_n(u)}{|u|}$ has been replaced by a more general condition. Besides, there have been some interests [12–14] in the existence of periodic

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