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Multiple-soliton solutions for extended (3 + 1)-dimensional Jimbo–Miwa equations

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ABSTRACT

In this work we investigate two extended (3+1)-dimensional Jimbo–Miwa equations. We use the simplified Hirota's method to derive multiple soliton solutions of distinct physical structures for each extended equation. We show that the dispersion relations and the phase shifts of the extended equations are distinct compared to the dispersion and shifts of the Jimbo–Miwa equation.

1. Introduction

The $(3+1)$-dimensional Jimbo–Miwa equation [1–11] reads

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0. \tag{1}$$

The Jimbo–Miwa equation (1) is the second equation in the well known KP hierarchy of integrable systems [1–4], which is used to describe certain interesting $(3+1)$-dimensional waves in physics, but does not pass any of the conventional integrability tests. The Jimbo–Miwa equation (1) was investigated regarding its solutions, non-integrability, and symmetries. This equation was investigated thoroughly in the literature where a variety of useful methods was invested in these works. The Painlevé method [3], the tanh–coth method [4,5,12–14], the simplified Hirota's method, the extended homoclinic test approach [6], a transformed rational function method [7], and other methods were applied to obtain solitons, periodic, complexiton, and travelling wave solutions. The $(3+1)$-dimensional Jimbo–Miwa equation is a rich model that gives a variety of exact solutions of distinct structures. It is the aim of this study to extend our work in [4] by introducing two extended $(3+1)$-dimensional forms of this equation.

In this work we propose two extended $(3+1)$-dimensional Jimbo–Miwa equations given as

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3(u_{xz} + u_{yz} + u_{zz}) = 0, \tag{2}$$

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and

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2(u_{xt} + u_{yt} + u_{zt}) - 3u_{xz} = 0, \tag{3}$$

where the last two linear terms u_{xz} and u_{yt} of (1) were extended to $u_{xz} + u_{yz} + u_{zz}$ and $u_{xt} + u_{yt} + u_{zt}$ respectively. The two extended forms retained the order and the dimension of the standard Jimbo–Miwa equation.

Searching for exact solutions of nonlinear equations is important in scientific and engineering applications because it offers a rich knowledge on the mechanism of the complicated physical phenomena modelled by these nonlinear equations. A set of systematic methods have been used in the literature to obtain for reliable treatments of nonlinear equations. A variety of powerful methods have been used to study the nonlinear evolution equations, such as the Hirota bilinear method [12–20], the Bäcklund transformation method, Darboux transformation, Pfaffian technique, the inverse scattering method, the Painlevé analysis, the generalized symmetry method and other methods. The inverse scattering method of integrable problems is more general than the Hirota's bilinear method which yields special solutions. The computer symbolic systems such as Maple and Mathematica allow us to perform complicated and tedious calculations.

The objectives of this work are twofold. First, we seek to study two extended Jimbo–Miwa equations (2) and (3) to establish multiple-soliton solutions of distinct physical structures when compared to the solutions of (1). We show that Eqs. (1)–(3) possess distinct dispersion relations, and distinct phase shifts which consequently will lead to distinct soliton solutions. The simplified Hirota's method [16–20] will be used to achieve this goal. We aim second to show that the power of the simplified Hirota's method is its ease of use when combined with other existing techniques such as the inverse scattering method.

To gain further insight into the three distinct equations (1)–(3), we briefly present the results we reported in [4] for the standard Jimbo–Miwa equation. This summary will be helpful for comparison with the results that will be obtained for the two extended equations.

2. Summary of the existing results

In [4], we substituted

$$u(x, y, z, t) = e^{k_i x + r_i y + s_i z - c_i t} \tag{4}$$

into the linear terms of (1) to find the dispersion relation as

$$c_i = \frac{k_i(k_i^2 r_i - 3s_i)}{2r_i}, \quad i = 1, 2, \dots, N, \tag{5}$$

and hence the wave variable θ_i becomes

$$\theta_i = e^{k_i x + r_i y + s_i z - \frac{k_i(k_i^2 r_i - 3s_i)}{2r_i} t}, \quad i = 1, 2, \dots, N. \tag{6}$$

We next used the substitution

$$u(x, y, z, t) = 2(\ln f)_x, \tag{7}$$

where

$$f(x, y, z, t) = 1 + e^{\theta_1}. \tag{8}$$

Consequently, the single soliton solution

$$u(x, y, z, t) = \frac{2k_1 e^{k_1 x + r_1 y + s_1 z - \frac{k_1(k_1^2 r_1 - 3s_1)}{2r_1} t}}{1 + e^{k_1 x + r_1 y + s_1 z - \frac{k_1(k_1^2 r_1 - 3s_1)}{2r_1} t}} \tag{9}$$

follows immediately.