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Piecewise shooting reproducing kernel method for linear singularly perturbed boundary value problems

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ABSTRACT

In this letter, a new numerical method is proposed for solving second order linear singularly perturbed boundary value problems with left layers. Firstly a piecewise reproducing kernel method is proposed for second order linear singularly perturbed initial value problems. By combining the method and the shooting method, an effective numerical method is then proposed for solving second order linear singularly perturbed boundary value problems. Two numerical examples are used to show the effectiveness of the present method.

1. Introduction

Consider a class of linear singular perturbation problems of the form

$$\begin{cases} \varepsilon u''(x) + a(x)u'(x) + b(x)u(x) = g(x), & 0 < x < 1, \\ u(0) = \alpha_0, \qquad u(1) = \alpha_1, \end{cases} \tag{1.1}$$

where $0 < \varepsilon \ll 1$, $a(x), b(x)$ and $g(x)$ are assumed to be sufficiently smooth, and such that (1.1) has a unique solution. Furthermore, we assume that $a(x) \geq \alpha > 0$, α is constant. The above assumption implies that the boundary layer of the solution to (1.1) will be in the neighborhood of $x = 0$.

Based on the reproducing kernel theory, a method called the reproducing kernel method (RKM) was developed by Cui, Geng et al. [1,2]. The method has been widely applied to many fields [3–25]. However, the direct application of the RKM to the singularly perturbed problems cannot produce accurate numerical solution due to the character of boundary layers. Recently, based on the proposed RKM, some effective numerical methods have been proposed for solving singularly perturbed turning point problems having twin

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boundary layers, singularly perturbed turning point problems with an interior layer and singularly perturbed delay initial and boundary value problems [20–25].

The rest of the paper is organized as follows. In the next section, the numerical method for solving (1.1) is introduced. The piecewise reproducing kernel method is proposed for second order linear singularly perturbed initial value problems in Section 3. The numerical examples are provided in Section 4. Section 5 ends this paper with a brief conclusion.

2. Method for singularly perturbed boundary value problem (1.1)

Due to the existence of boundary layers, we hope to solve (1.1) by using the RKM in a piecewise fashion. However, it is difficult to solve (1.1) directly in this way. Since it is possible to solve singularly perturbed initial value problems by piecewise reproducing kernel method, by the idea of shooting method, one natural way to attack (1.1) is to solve the related singularly perturbed initial value problem with a guess as to the appropriate initial value.

Consider the following two singularly perturbed initial value problems related to (1.1)

$$\begin{cases} \varepsilon v''(x) + a(x)v'(x) + b(x)v(x) = g(x), \quad 0 < x < 1, \\ v(0) = \alpha_0, \qquad v'(0) = z_1. \end{cases} \tag{2.1}$$

and

$$\begin{cases} \varepsilon w''(x) + a(x)w'(x) + b(x)w(x) = g(x), \quad 0 < x < 1, \\ w(0) = \alpha_0, \qquad w'(0) = z_2. \end{cases} \tag{2.2}$$

Suppose we have solved (2.1) and (2.2) and obtained their solutions $v(x)$, $w(x)$ respectively. Now we form a linear combination of $v(x)$ and $w(x)$:

$$u(x) = \lambda v(x) + (1-\lambda)w(x), \tag{2.3}$$

where λ is a parameter to be determined. Clearly, $u(x)$ solves Eq. (1.1) and meets the first of two boundary conditions, that is, $u(0) = \alpha_0$. Selecting λ so that $u(1) = \alpha_1$, we have

$$u(1) = \lambda v(1) + (1-\lambda)w(1) = \alpha_1$$

and

$$\lambda = \frac{\alpha_1 - w(1)}{v(1) - w(1)}. \tag{2.4}$$

Theorem 2.1. *If problem (1.1) has a solution, then either $v(x)$ itself is a solution or $v(1) - w(1) \neq 0$ ($u(x)$ is a solution).*

Proof. Let $y_0(x), y_1(x)$ and $y_2(x)$ be solutions of the following initial value problems, respectively

$$\begin{cases} \varepsilon y_0''(x) + a(x)y_0'(x) + b(x)y_0(x) = g(x), \quad 0 < x < 1, \\ y_0(0) = \alpha_0, \qquad y_0'(0) = 0, \end{cases} \tag{2.5}$$

$$\begin{cases} \varepsilon y_1''(x) + a(x)y_1'(x) + b(x)y_1(x) = 0, \quad 0 < x < 1, \\ y_1(0) = 1, \qquad y_1'(0) = 0, \end{cases} \tag{2.6}$$

and

$$\begin{cases} \varepsilon y_2''(x) + a(x)y_2'(x) + b(x)y_2(x) = 0, \quad 0 < x < 1, \\ y_2(0) = 0, \qquad y_2'(0) = 1. \end{cases} \tag{2.7}$$