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A simple monodimensional model coupling an enthalpy transport equation and a neutron diffusion equation

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ABSTRACT

We obtain an analytic solution of a monodimensional stationary system coupling a simplified thermohydraulic model to a simplified neutronic model based on the diffusion approximation with one energy group. We obtain this solution with minimal hypotheses on the absorption and fission cross sections, and on the diffusion coefficient.

1. Introduction

In this Note, we construct an analytic solution of the low Mach number thermohydraulic model

$$\begin{cases} \dfrac{d}{dz}(\rho u) = 0, & \text{(a)} \\ \dfrac{d}{dz}(\rho u^2 + \pi) = \rho g, & \text{(b)} \\ \rho u \dfrac{d}{dz} h = \mathbb{E}\Sigma_f(h)\phi(t,z) & \text{(c)} \end{cases} \tag{1}$$

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coupled to the simplified neutronic model based on the diffusion approximation with one energy group

$$-\frac{d}{dz}\left[D(h)\frac{d}{dz}\phi(z)\right]+\left[\Sigma_a(h)-\frac{\nu\Sigma_f(h)}{k_{\mathit{eff}}}\right]\phi(z)=0. \tag{2}$$

In (1) and (2), $z\in[0,L]$ is the spatial variable, $L>0$ being the length of the nuclear core. Moreover, in (1), $\rho(z)$, $u(z)$, $\pi(z)$ and $h(z)$ are respectively the density, the velocity, the dynamical pressure and the internal enthalpy of the flow. The source term ρg is a volumic force (*e.g.* the gravity field). The constant $\mathbb{E}$ is the energy released by a fission ($\mathbb{E}>0$ is in Joule), $\Sigma_f(h)$ is the fission (macroscopic) cross section ($\Sigma_f(h)>0$ is in m^{-1}) and $\phi(z)$ – solution of (2) – is the scalar neutron flux ($\phi(z)\geq 0$ is in $\mathrm{m}^{-2}\ \mathrm{s}^{-1}$). In (2), $D(h)$ is the diffusion coefficient ($D(h)>0$ is in m), $\Sigma_a(h)$ is the absorption (macroscopic) cross section ($\Sigma_a(h)>0$ is in m^{-1}) and ν is the average number of neutron produced by a fission. Moreover, the density ρ and the internal enthalpy h are linked through the equation of state $\rho=\varrho(h)$ where $\varrho(\cdot)$ is a given function.[1] At last, $k_{\mathit{eff}}>0$ is the neutron multiplication factor: $k_{\mathit{eff}}\in]0,1[$, $k_{\mathit{eff}}=1$ and $k_{\mathit{eff}}>1$ means that the nuclear core is respectively subcritical, critical and supercritical.

Using (1), we obtain $\rho u=D_e$ where $D_e>0$ is a positive constant defining the flow rate. Thus, (1)(c) and (2) give the simplified thermohydraulics–neutronics system

$$\begin{cases} D_e\dfrac{dh}{dz}=\mathbb{E}\Sigma_f(h)\phi, & \text{(a)}\\ -\dfrac{d}{dz}\left[D(h)\dfrac{d\phi}{dz}\right]+\left[\Sigma_a(h)-\dfrac{\nu\Sigma_f(h)}{k_{\mathit{eff}}}\right]\phi=0. & \text{(b)}\end{cases} \tag{3}$$

We supplement this system, written for $\phi\in H_0^1([0,L])$ (which means that ϕ satisfies homogeneous Dirichlet boundary conditions $\phi(0)=\phi(L)=0$)[2] and $h\in C^1([0,L])$, with the constraint $\phi\geq 0$ on $[0,L]$ and with the boundary conditions

$$h(0)=h_e \quad \text{and} \quad h(L)=h_s. \tag{4}$$

Let us note that knowing $h(z)$, the density $\rho(z)$ is given by $\rho(z)=\varrho[h(z)]$. This allows to obtain the velocity $u(z)$ with $u(z)=\frac{D_e}{\rho(z)}$. At last, the dynamical pressure $\pi(z)$ is obtained by integrating (1)(b) and by using the boundary condition $\pi(L)=\pi_*$ where π_* is the pressure at the outlet of the nuclear core. In [2], we construct an analytical solution of (3) (4) when $D(h)$ and $\Sigma_f(h)$ are positive constants, $\Sigma_a(h)$ being a non-constant function of h (to enforce the coupling). In this Note, we generalize this result by supposing that $D(h)$ and $\nu\Sigma_f(h)$ are also functions depending on h.

The outline of this Note is the following. In Section 2, we construct an analytical solution of (3) (4). In Section 3, we underline the link between (3) (4) and an eigenvalue problem. Then, we conclude the Note.

2. Construction of an analytical solution

To construct an analytical solution of (3) (4), we assume that the given functions $\Sigma_f(h)$, $\Sigma_a(h)$ and $D(h)$ verify the two following hypotheses:

Hypothesis 1. The enthalpy always belongs to a fixed domain $[h_{\min},h_{\max}]$ on which $\Sigma_f(h)$, $\Sigma_a(h)$ and $D(h)$ are continuous functions.

Hypothesis 2. There exist $\alpha_f>0$, $\alpha_a>0$ and $\alpha_d>0$ such that in $[h_{\min},h_{\max}]$

$$\Sigma_f(h)\geq\alpha_f, \qquad \Sigma_a(h)\geq\alpha_a \quad \text{and} \quad D(h)\geq\alpha_d. \tag{5}$$

[1] The fact that the equation of state $\varrho(h)$ depends only on h is a consequence of the low Mach regime [1].

[2] This is the natural set-up for this second order elliptic equation, which is thus written in the weak sense.