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Global dynamics of a predator–prey model with defense mechanism for prey[☆]

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ABSTRACT

Recently, Venturino and Petrovskii proposed a general predator–prey model with group defense for prey species (Venturino and Petrovskii, 2013). The local dynamics had been studied and showed that the model might undergo Hopf bifurcation, and have an extinction domain. In this paper, we dedicate ourselves to the investigation of the global dynamics of the model by establishing the conditions of the nonexistence of periodic orbits, and the existence and uniqueness of limit cycles.

1. Introduction

The rate of change of prey attacked, in population dynamics, is generally described by the so-called functional response of the predator to the prey [1]. By the mass action principle and assuming that both predator and prey are of spatially homogeneous distribution, we can obtain the traditional functional response, which is based on the original L–V model and still remained as the basis of theory for predator–prey problems [2].

For the inhomogeneous situation, Venturino and Petrovskii [3] recently proposed a functional response in terms of the α power of prey, where $\alpha \in (0, 1)$ reflecting group defense for prey species. More precisely, they investigated the following model:

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$$\begin{cases} \dfrac{\mathrm{d}G(\tau)}{\mathrm{d}\tau} = r\left(1 - \dfrac{G(\tau)}{K}\right) G(\tau) - aG(\tau)^{\alpha} F(\tau), \\ \dfrac{\mathrm{d}F(\tau)}{\mathrm{d}\tau} = -mF(\tau) + aeG(\tau)^{\alpha} F(\tau), \end{cases} \tag{1.1}$$

with all parameters are positive and functions $G(\tau)$ and $F(\tau)$ denote respectively the densities of prey and predator species at time τ. r is the growth rate of prey species, K is its carrying capacity, m is the mortality rate of predator species, a is the search efficiency of predator for prey, e is the biomass conversion coefficient, and α represents a kind of aggregation efficiency. Then the local dynamics was studied, such as Hopf bifurcation and existence of extinction domain. Two special defense mechanisms have also been considered by other researchers, such as in Ref. [4], Braza considered a special predator–prey system, in which the prey population exhibits herd behavior in order to provide a self-defense from predators while the predator population shows individualistic behavior, and modeled the functional response in terms of the 1/2 power of prey. The author uncovered that the model can experience Hopf bifurcation for some parameters, and some singularity occurs for the solution behavior near the trivial equilibrium. In Ref. [5], Chattopadhyay and his coauthors considered two toxin-producing phytoplankton–zooplankton systems in which a fraction of the phytoplankton aggregates to form some roughly spherical patches, and the poison will leak into the surrounding water through the surface of the patch. They modeled the poisoning function in terms of the 2/3 power of phytoplankton, and obtained that the dynamics of the plankton population depends on the fraction of the phytoplankton population that aggregates to form patches. We refer readers to Refs. [6–10] as some other related works on predator–prey model with herd behavior.

It is worth pointing out that, in all of these studies, authors only investigated local dynamics such as stability of equilibria; the global dynamics, however, was missing. In the present paper, we will devote ourselves to the investigation of the global dynamics of model (1.1). We will first perform some basic analysis for model (1.1), including the singularity analysis of the solution near the origin equilibrium, the stability analysis of the non-trial equilibria and the existence of Hopf bifurcations, which can be considered as the natural extensions of the results known in Refs. [3–5]. Then the conditions under which model (1.1) has no periodic orbits, and has exactly one limit cycle will be established. Based on these results, we can obtain the global dynamics of model (1.1) as follows: there exists a separatrix in the positive invariant set $\mathbb{R}_+^2$ such that the orbit with initial value above the separatrix terminates at positive vertical axis after which decreases to zero along the vertical axis, and the orbit starting with a point below the separatrix converges to the corresponding attractor.

The rest of this paper is organized as follows. Some preliminary results of model (1.1) are discussed in Section 2. The conditions of the nonexistence of periodic orbits, and the existence and uniqueness of limit cycles are respectively established in Sections 3 and 4. We then present the global dynamics of model (1.1) in Section 5. Finally, we conclude the paper by a short discussion.

2. Some preliminary results

First, we introduce transformations

$$x = \frac{1}{K}G, \qquad y = \frac{a}{rK^{1-\alpha}}F, \qquad t = r\tau, \qquad c = \frac{aeK^{\alpha}}{r}, \qquad s = \frac{m}{r}$$

to reduce the number of parameters and to simplify the original model (1.1). After some straightforward calculation and manipulation we then reach a non-dimensional model

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = x(1-x) - x^{\alpha} y, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = cx^{\alpha} y - sy. \end{cases} \tag{2.1}$$