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Permanence and global attractivity of an impulsive delay Logistic model

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ABSTRACT

In this paper, we consider an impulsive delay Logistic model. First by mathematical analysis, we obtain the maximum and minimum values of solutions of the corresponding autonomous Logistic model. Then by applying the comparison theorem and constructing some suitable Lyapunov functionals, we discuss the permanence and the global attractivity of the model, based on the boundedness of solutions of the corresponding autonomous Logistic model. An example together with its numerical simulation is given to verify our main result.

1. Introduction

Note that processes of models whose motions depend on the history as well as undergo abrupt changes in their states are best described by delay differential equations. In 1948, [1] first proposed the following equation

$$\dot{x}(t) = rx(t)\Big(1 - \frac{x(t-\tau)}{K}\Big),$$

which is called the Hutchinson equation and regarded as a fundamental equation of mathematical ecology. Many papers have been dedicated to the study of solutions of this equation (see [2] for example).

However, it has been recognized that impulsive delay differential equations provide an adequate mathematical description for many real world phenomena [3–9]. So in this paper, we investigate the following impulsive delay Logistic model

$$\begin{aligned} &\dot{x}(t) = x(t)[a(t) - b(t)x(t-\tau)], \quad t \neq t_k, \\ &x(t_k^+) = h_k x(t_k), \quad k = 1, 2, \ldots, \end{aligned} \tag{1.1}$$

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with

$$x(t) = \varphi(t), \quad \text{for } -\tau \le t \le 0,\ \varphi \in L([-\tau, 0], [0, +\infty)),\ \varphi(0) > 0, \tag{1.2}$$

where $\tau > 0$ is the gestation period and $L([-\tau, 0],\ [0, +\infty))$ denotes the set of Lebesgue measurable functions on $[-\tau,\ 0]$; $x(t)$ is population density at time t; $a(t)$ and $b(t)$ are bounded and continuous functions on $(0, +\infty)$ with $b(t) \ge 0$ for all $t \in (0, +\infty)$; $0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$ are impulse points with $\lim_{k\to+\infty} t_k = +\infty$; the impulsive perturbations h_k $(k = 1, 2, \ldots)$ are positive.

When $\tau = 0$, [10] considered the periodic model (1.1) and obtained sufficient conditions for the existence and asymptotic stability of T-periodic solution; [11] also studied its global behaviors but under the condition that the first and second equations of the model have different periods; [12] further investigated the almost periodic solution of the almost periodic model (1.1). For the non-periodic model (1.1) with $\tau = 0$, [13] discussed its permanence and global stability, where they only proved that solutions of the system are bounded above and below by positive constants but failed to give the value of these constants. However, in a lot of practical problems, we are required to give the maximum and minimum values of solutions. Based on this, [14] tried to give the bounds of this system but whose result only holds under the assumption $0 < h_k < 1$, which is inconsistent with its condition $h_k > 0$. This motivates us to propose the first issue of this paper.

Issue 1: For system (1.1) with $\tau = 0$, is it possible for us to obtain the maximum and minimum values of solutions?

When $\tau > 0$, [15] discussed the existence of positive periodic solution of the periodic model (1.1) by the relation between the solutions of impulsive system and the corresponding non-impulsive system. Note that the dynamics of species is affected by both natural factors and human exploitation activities, but a lot of those elements are not periodic, so it is necessary for us to study the non-periodic model (1.1). Thus we propose the second issue of this paper.

Issue 2: How about the dynamical behaviors such as permanence and global attractivity of system (1.1) with all its coefficients being not periodic?

For any given continuous function $f(t)$, let f_L and f_M denote $\inf_{0\le t<+\infty} f(t)$ and $\sup_{0\le t<+\infty} f(t)$, respectively. For any sequence $\{h_k\}$, let h_L and h_M denote $\inf_{k\in Z} h_k$ and $\sup_{k\in Z} h_k$, respectively. Denote $\sup_{k\in Z} t_k^1 = \sup_{k\in Z}(t_{k+1} - t_k) = \eta$ and $\inf_{k\in Z} t_k^1 = \theta$. Obviously $\eta \ge \theta > 0$.

The organization of this paper is as follows. In Section 2, we study the permanence and global attractivity of the corresponding autonomous Logistic model of (1.1) with $\tau = 0$, which solves Issue 1. In Section 3, we investigate the permanence and global attractivity of system (1.1), which answers Issue 2. An example with its numerical simulation is given in Section 4.

2. Autonomous logistic system

In this section, we consider the following autonomous logistic system

$$\begin{aligned} \dot{x}(t) &= x(t)(a - bx(t)), \quad t \ne t_k, \\ x(t_k^+) &= h_k x(t_k), \quad k = 1, 2, \ldots, \end{aligned} \tag{2.1}$$

where a and b are positive constants.

Lemma 2.1. *Suppose that:*

$$a\theta + \ln h_L > 0. \tag{2.2}$$

Then: (a) *For any positive solution $x(t)$ of system (2.1), $m_0 \le \liminf_{t\to\infty} x(t) \le \limsup_{t\to\infty} x(t) \le M_0$, where*

$$M_0 = \min\Big\{\frac{a\eta + \ln h_M}{b\eta h_M}, \frac{(a\theta + \ln h_M)h_M}{b\theta}\Big\} \triangleq \min\{M_0^{(1)}, M_0^{(2)}\},$$