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A new result on finite-time control of singular linear time-delay systems

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A B S T R A C T

In this paper, problem of robust finite-time stability and control is first time discussed for singular linear time-delay systems subject to disturbance. By developing delay singular value decomposition approach combining with linear matrix inequality (LMI) technique, new sufficient conditions for the existence of such controllers are proposed in terms of the solvability to a set of LMIs. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction and problem formulation

In this paper we are concerned with the following singular linear time-delay system

$$\begin{cases} E\dot{x}(t) = Ax(t) + Dx(t-h) + Bu(t) + B_1w(t), \quad t \geq 0, \\ x(t) = \psi(t), \quad \forall t \in [-h, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^{m_1}$ is the control, $w(t) \in R^{m_2}$ is the disturbance; A, D, B, B_1, are constant matrices of appropriate dimensions; $E \in R^{n\times n}$ is a singular matrix, rank $E = r < n$. This system is viewed as a singular linear time-delay system, which has attracted particular interest in the literature due to the comprehensive applications in economics, robotics, electrical and chemical systems [1,2]. The research activities in stability and control of singular systems with delay have provided many interesting results using algebraic methods, state-space decomposition approaches, Lyapunov functional method, etc. [3–6]. On the other hand, in many practical applications, the main concern is the behavior of the system

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within a finite horizon, such as aerospace control systems, machining control systems, etc., the control processes of which are implemented in a finite-time interval. To deal with such situations, the problem of finite-time stability (FTS) focuses its attention on the behavior of a system response over a finite-time interval [7]. FTS means that once we fix a time interval, the state of a system does not exceed a certain bound during this specified time interval. It is important to recall that FTS and Lyapunov asymptotic stability (LAS) are independent concepts; indeed a system can be FTS but not LAS, and vice versa. While LAS deals with the behavior of a system at infinity time, FTS is a more practical concept, useful to study the behavior of the system within a finite short interval, and therefore it finds useful applications whenever it is desired that the state variables do not exceed a given threshold during the transients. The finite-time stabilization concerns with the design of a feedback controller which ensures the FTS of the closed-loop system and the problem of guaranteed cost control (GCC) is to find a feedback controller to finite-time stabilize the system guaranteeing an adequate cost level of performance. Based on linear matrix inequality techniques, some results have been obtained in [8, 9] for FTS and GCC of linear time-delay systems. However, when the controller is designed for a real plant, it is also desirable to design a controller that not only makes the closed-loop singular system asymptotically stable, but also guarantees an adequate level of performance [10–12]. It should be noticed that all the related results on stability and control for singular systems mentioned above were developed in the context of Lyapunov stability, while very little attention has been paid to the finite-time stability. The main contribution of this paper is to get new sufficient conditions for the design of a state feedback controller which makes the closed-loop system finite-time stable and guarantees an adequate cost level of performance.

Definition 1.1. (i) System (1) is regular if $det(sE - A)$ is not identical zero. (ii) System (1) is impulse-free if $deg(det(sE - A)) = r = rankE$.

The singular delay system (1) may have an impulsive solution, however, the regularity and the absence of impulses of the pair (E, A) ensure the existence and uniqueness of an impulse free solution to this system, which is shown in [1].

Definition 1.2. For given positive numbers c_1, c_2, T and a symmetric positive definite matrix $\mathbb{R}$, the singular system (1) is robustly finite-time stabilizable w.r.t $(c_1, c_2, T, \mathbb{R})$ if it is regular, impulse-free and there exists a feedback control $u(t) = Kx(t)$ such that the solution of the closed-loop system $E\dot{x}(t) = (A + BK)x(t) + Dx(t - h) + B_1 w(t)$ satisfies the following relation

$$\sup_{-h\le s\le 0}\{\psi^\top(s)\mathbb{R}\psi(s)\} \le c_1 \Longrightarrow x^\top(t)\mathbb{R}x(t) < c_2, \quad \forall t \in [0, T],$$

for all disturbances $w(t)$ satisfying $w^\top(t)w(t) \le d$, $t \in [0, T]$, for a given number $d > 0$.

Definition 1.3. If there exist a feedback control law $u^*(t) = Kx(t)$ and a positive number J^* such that the system (1) is robustly finite-time stabilizable w.r.t $(c_1, c_2, T, \mathbb{R})$ and the cost function satisfies $J(u^*) \le J^*$, where

$$J(u) = \int_0^T [x^\top(t)Q_1x(t) + x^\top(t - h)Q_2x(t - h) + u^\top(t)Q_3u(t)]dt, \tag{2}$$

$Q_1, Q_2 \in R^{n\times n}$ are symmetric non-negative definite matrices, $Q_3 \in R^{m_1\times m_1}$ is a symmetric positive definite matrix, then the value J^* is a guaranteed cost value and the designed control $u^*(t)$ is said to be a guaranteed cost controller.

The problem is to design a state feedback controller such that the closed-loop system is finite-time stable and the cost function value is less than a specified upper bound for all admissible disturbances.