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Near-field imaging with far-field data

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ABSTRACT

Using the inverse diffractive grating problem as an example, we demonstrate how a super-resolution can be achieved stably by using far-field data. The idea is to place a slab of a homogeneous medium with a large index of refraction above the grating surface, and more propagating wave modes can be utilized from the far-field data which contributes to the reconstruction resolution.

1. Introduction

According to the Rayleigh criterion, there is a resolution limit to the sharpness of details that can be observed by conventional far-field optical microscopy, one half the wavelength, referred to as the diffraction limit [1]. The loss of the details is related to the non-radiative components of the field known as evanescent waves [2]. It is severely ill-posed to directly use the evanescent waves since the noise in the measurements will be amplified exponentially. Therefore, near-field data is of paramount importance to achieve super-resolution [3,4]. However, it might be cumbersome to measure the near-field data as a sophisticated control is needed for the probe when scanning samples.

We use the diffraction grating problem as an example to demonstrate how a super-resolution can be achieved stably by using the far-field data. The idea is to place a slab of a homogeneous medium with a large index of refraction above the grating surface. A particular function of the slab is to convert more propagating wave modes of the far-field data into the near-field. The approach avoids measuring the sensitive near-field data.

Scattering theory in periodic structures has many significant applications in optical industry. The scattering problems have been studied extensively for periodic structures [5–9,14,15]. This paper is built upon

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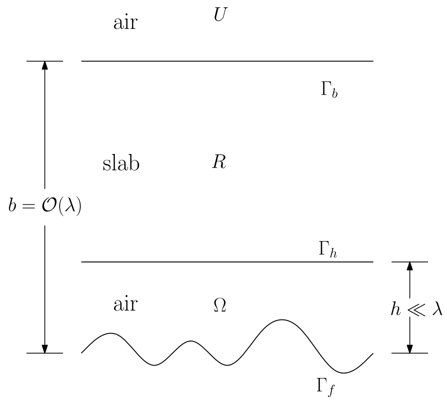

Fig. 1. Schematic of the problem geometry.

our recent work on solving a wide class of inverse surface scattering problems for acoustic, electromagnetic, and elastic waves [10–12], where the methods were designed especially for the near-field data. It reflects our effort to design more practical models and efficient methods for solving quantitatively complex inverse scattering problems with high resolution.

2. Problem formulation

Consider a perfect electrically conducting surface $\varGamma_f = \{(x, y) \in \mathbb{R}^2 : y = f(x), 0 < x < \varLambda\}$, where f is a periodic function with period $\varLambda$. The scattering surface function f is assumed to have the form

$$f(x) = \varepsilon g(x), \tag{2.1}$$

where $\varepsilon > 0$ is a sufficiently small constant g is also a periodic function with the same period $\varLambda$. Hence the surface $\varGamma_f$ is a small perturbation of the planar surface $\varGamma_0 = \{(x, y) \in \mathbb{R}^2 : y = 0,\ 0 < x < \varLambda\}$.

Let a slab of a homogeneous dielectric medium be placed above $\varGamma_f$. The slab's bottom face is $\varGamma_h = \{(x, y) \in \mathbb{R}^2 : y = h,\ 0 < x < \varLambda\}$, where h, satisfying $\|f\|_\infty < h \ll \lambda$, is a positive constant. Here λ is the wavelength of the incident field. The slab's top face is $\varGamma_b = \{(x, y) \in \mathbb{R}^2 : y = b,\ 0 < x < \varLambda\}$, where b, satisfying $h \ll b = \mathcal{O}(\lambda)$, is also a positive constant.

Denote by $\varOmega$ the bounded domain between $\varGamma_f$ and $\varGamma_h$, i.e., $\varOmega = \{(x, y) \in \mathbb{R}^2 : f < y < h,\ 0 < x < \varLambda\}$. Let R be the domain of the slab, i.e., $R = \{(x, y) \in \mathbb{R}^2 : h < y < b,\ 0 < x < \varLambda\}$. Denote by U the open domain above $\varGamma_b$, i.e., $U = \{(x, y) \in \mathbb{R}^2 : y > b,\ 0 < x < \varLambda\}$. The index of refraction is one in $\varOmega$ and U since they are free spaces, and has a constant value $n > 1$ in the slab R. The schematic of the problem geometry is shown in Fig. 1.

Let an incoming plane wave $\phi^{\mathrm{inc}}(x, y) = e^{-\mathrm{i}\kappa y}$ be normally incident on $\varGamma_b$ from above, where κ is the free space wavenumber. Let ψ, ϕ, and φ be the diffracted field in U, the total field in R, and the total field in $\varOmega$, respectively. They satisfy the Helmholtz equations:

$$\begin{cases} \varDelta\psi + \kappa^2\psi = 0 & \text{in } U, \\ \varDelta\phi + (\kappa n)^2\phi = 0 & \text{in } R, \\ \varDelta\varphi + \kappa^2\varphi = 0 & \text{in } \varOmega, \end{cases} \tag{2.2}$$

and the boundary conditions:

$$\begin{cases} \varphi = 0 & \text{on } \varGamma_f, \\ \partial_y\psi = \mathscr{B}\psi & \text{on } \varGamma_b. \end{cases} \tag{2.3}$$