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The a posteriori error estimates of Chebyshev–Petrov–Galerkin methods for second-order equations[☆]

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ABSTRACT

In this paper, the a posteriori error estimates of Chebyshev–Petrov–Galerkin approximations are investigated. For simplicity, we choose the Poisson equation with Dirichlet boundary conditions to discuss the a posteriori error estimators, and deduce their efficient and reliable properties. Some numerical experiments are performed to verify the theoretical analysis for the a posteriori error estimators.

1. Introduction

The spectral methods are extensively used to solve partial differential equations using Legendre, Chebyshev and ultraspherical polynomials, see [1–9] and the references cited therein. Legendre–Galerkin methods guarantee symmetry and sparsity properties of the stiffness matrices for elliptic boundary value problems, i.e., for the Poisson equations, the corresponding stiffness matrix is reduced to an identity matrix. However, due to the nonuniform weight associated with the Chebyshev polynomials, the matrices of resulting discretized systems reduce to pentadiagonal matrices [5]. The key advantage of using the Chebyshev polynomials is that the discrete Chebyshev transforms can be performed within $O(Nlog_2N)$ operations by using fast Fourier transform, but the discrete Legendre transforms are $O(N^2)$ [5].

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Generally, in order to get a numerical solution with acceptable accuracy, one may enhance the numbers of basis or refine the mesh if the a posteriori error indicators are bigger than some given criteria, (for example, see [10,11]). However, the a posteriori error estimates for spectral methods have been much less developed. There are only few papers on this topic in current literatures, see [12–17] and the references therein. Guo [14] summarized the theoretical results of a posteriori error estimations for high order finite element methods. The authors [17] gave an improved a posteriori error estimator with explicit scheme for Legendre–Galerkin spectral method in one dimension.

To avoid deriving the equivalent variational formulation, one may directly integrate the original equations with test variables in the domain. For this aim, one needs to choose different discretized function spaces for the trial function and test function. It is well known that the test function space is substantially different from the trial function space for a Petrov–Galerkin framework, which is the key difference from that in [17]. And this method may lack the symmetry of the second-order problems.

Following this point, Chebyshev–Petrov–Galerkin (CPG, for short) approximations efficiently provide higher accuracy for the engineering applications, see [1–3,5]. Specially, the author in [3] employed the kth derivatives of Chebyshev polynomials to present an efficient CPG spectral method for solving second-order equations, and constructed the corresponding basis and test functions. In lots of engineering applications, based on the a posteriori error estimators, one readily determines the minimum calculation scale and the more economic computational cost. To the best of authors' knowledge, there are fewer works on the a posteriori error estimates of CPG in the literature.

The purpose of this paper is to investigate the a posteriori error indicators for CPG spectral methods with the orthogonal properties of Chebyshev polynomials. This work focuses on the model problems in an interval. The analyses show that the efficient and reliable properties of the given a posteriori error estimators hold, which can be easily used in engineering applications.

The outline of this article is as follows. In Section 2, we state Poisson equation with Dirichlet boundary conditions in one dimension and recall some well-known results about Chebyshev polynomials, furthermore, we state the discrete schemes of the model with CPG spectral methods. In Section 3, we derive some a posteriori error estimators and prove their efficiency and reliability. In Section 4, some numerical examples are given to confirm the theoretical results given in above sections. Finally, the conclusions and future work are briefly listed in Section 5.

2. The model problem and its discretized approximations

We give some basic notations which will be used in the sequel. Let $\Omega = (-1, 1)$ with the boundary set $\partial\Omega = \{-1, 1\}$. Throughout this paper we use the standard notations of Sobolev spaces [18]. We shall use c and C to denote some generic positive constants independent of any function and the degree of polynomials. Without loss of generality, we consider the following typical Poisson equation with homogeneous Dirichlet boundary condition

$$\begin{cases} -u'' = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{2.1}$$

As we all known, this equation has a unique solution. We denote the first kind Chebyshev polynomials by $T_k(x)$, i.e.,

$$T_0 = 1, \qquad T_1 = x, \qquad T_{j+1} = 2xT_j - T_{j-1}, \quad j = 1, 2, \ldots.$$