Applied Mathematics Letters 61 (2016) 8–12

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Breather-to-soliton conversions and nonlinear wave interactions in a coupled Hirota system

Hui-Hui Zhao, Xiao-Juan Zhao, Hui-Qin Hao*

School of Mathematics, Taiyuan University of Technology, Taiyuan 030024, China

ARTICLE INFO

Article history:
Received 17 April 2016
Received in revised form 28 April 2016
Accepted 28 April 2016
Available online 7 May 2016

Keywords:
Breather–soliton dynamics
A coupled Hirota system
Multi-peak soliton
Antidark soliton
Periodic wave

ABSTRACT

In this paper, a coupled Hirota system with higher-order effects is analytically investigated. The results show that the breather solutions can be converted into some types of nonlinear localized and periodic solutions on the plane-wave background. The exact relations for the conversions are presented, which depend on the higher-order effects, the background frequency and the eigenvalue. Via some graphic illustrations, the collisions between these nonlinear waves in the second-order conversions are displayed.

1. Introduction

Recently, the inherent relations between the breather and soliton solutions have been investigated in several nonlinear evolution equations [1–6]. Under certain conditions, breathers can be converted into some types of localized and periodic waves, such as multi-peak soliton, antidark soliton, W-shaped soliton and periodic wave. Specifically, only when the equation possesses one or more free parameters can this transition happen.

The possibility of a breather decaying into solitons in the nonlinear Schrödinger (NLS) equation has been demonstrated in Ref. [1]. By the modulational instability (MI), the results have indicated that the standard NLS cannot provide this process. Ref. [2] has revealed that many localized and periodic waves can be extracted from a unified exact solution under specific parameter conditions in the coupled NLS–MB equations. For the Hirota equation, Ref. [3] has presented an exact expression for the transformation. Furthermore, the state transition between the rogue wave and W-shaped wave has also been analyzed in Ref. [6]. In this paper, we consider the conversion in a coupled Hirota system

* Corresponding author.
E-mail address: math0351@sina.com (H.-Q. Hao).

http://dx.doi.org/10.1016/j.aml.2016.04.016

$$u_z = i\left[\frac{1}{2}u_{tt} + (|u|^2+|v|^2)u + \epsilon\,[u_{ttt} + 3\,(2|u|^2+|v|^2)u_t + 3\,u\,v^* v_t]\right], \tag{1.1a}$$

$$v_z = i\left[\frac{1}{2}v_{tt} + (|u|^2+|v|^2)v + \epsilon\,[v_{ttt} + 3\,(2|u|^2+|v|^2)v_t + 3\,v\,u^* u_t]\right], \tag{1.1b}$$

where, $u(z,t)$ and $v(z,t)$ are the complex smooth envelop functions, z and t represent the longitudinal distance and retarded time respectively, and ϵ denotes the coefficient of higher-order effects including the third-order dispersion, self-steepening and Raman scattering terms. The main aim of this paper is to construct the breather-to-soliton conversions, derive several kinds of localized and periodic waves, and to analyze the characteristics of the interactions among these nonlinear waves in System (1.1).

2. One-order breather-to-soliton conversions

Based on some known results of the Darboux transformation (DT) for System (1.1) [7,8], the new solutions in the first iteration can be given by

$$u^{[1]} = u^{[0]} - \frac{2\,i(\lambda_1-\lambda_1{}^*)\phi_1\,\phi_2{}^*}{|\phi_1|^2+|\phi_2|^2+|\phi_3|^2}. \tag{2.1}$$

$\Phi_1 = (\phi_1,\phi_2,\phi_3)^T$ is a special solution corresponding to the spectral parameter λ_1. The component v in System (1.1) can be discussed in the same way. Choosing $u^{[0]} = v^{[0]} = u_0\,e^{i(k\,z+w\,t)}$ and separating the real and imaginary part of the variables and functions involved in the above expression, we can extend the one-breather solutions on the plane-wave background as

$$u^{[1]} = u_0\left(1 + 4\,b\,\frac{N_r + i\,N_i}{D}\right)e^{i\,(k\,z+w\,t)}, \tag{2.2}$$

where

$$\begin{aligned}
&\kappa = \kappa_r + i\,\kappa_i = \sqrt{2\,u_0^2 + (\lambda_1 + w/2)^2}, \qquad \eta_1 = (a+w/2)^2 + b^2 + 2\,u_0^2,\\
&\eta_2 = 2\,(a\,\kappa_i - b\,\kappa_r) + w\,\kappa_i, \qquad \eta_3 = 2\,(a\,\kappa_r + b\,\kappa_i) + w\,\kappa_r, \qquad \lambda_1 = a + i\,b,\\
&N_r = -b\,\cos[2\,A_r] - b\,\cosh[2\,A_i] - \kappa_i\,\sinh[2\,A_i] + \kappa_r\,\sin[2\,A_r],\\
&N_i = (a+w/2)\,\cos[2\,A_r] + (a+w/2)\,\cosh[2\,A_i] + \kappa_i\,\sin[2\,A_r] + \kappa_r\,\sinh[2\,A_i],\\
&D = (\eta_1 - |\kappa|^2)\cos[2\,A_r] + (\eta_1 + |\kappa|^2)\cosh[2\,A_i] + \eta_2\,\sin[2\,A_r] + \eta_3\,\sinh[2\,A_i],
\end{aligned}$$

with

$$\begin{aligned}
&A_r = t\,\kappa_r + z\,V_r, \qquad A_i = t\,\kappa_i + z\,V_i,\\
&V_r = ((a-w/2)(1+2\,w\,\epsilon) + 4\,(u_0^2 - a^2 + b^2)\,\epsilon)\,\kappa_r - b\,(1 - 8\,a\,\epsilon + 2\,w\,\epsilon)\,\kappa_i,\\
&V_i = ((a-w/2)(1+2\,w\,\epsilon) + 4\,(u_0^2 - a^2 + b^2)\,\epsilon)\,\kappa_i + b\,(1 - 8\,a\,\epsilon + 2\,w\,\epsilon)\,\kappa_r.
\end{aligned}$$

Solution (2.2) contains two kinds of functions: the trigonometric functions and the hyperbolic functions, which describe the propagation of periodic waves and localized solitons respectively. V_i/κ_i and V_r/κ_r are the corresponding velocities. The transformation of breather solutions into solitons becomes possible when the velocities are identical, that is $V_i/\kappa_i = V_r/\kappa_r$. Insertion of the detailed V_i and V_r into it yields the exact relation as

$$a = \frac{1+2\,w\,\epsilon}{8\,\epsilon}. \tag{2.3}$$

Relation (2.3) includes the higher-order coefficient ϵ, the real parameter a and the background frequency w. ϵ is necessary for this conversion, and its value cannot be zero. In Fig. 1, we convert a breather into the non-periodic solitons for System (1.1). Fig. 1(a) depicts a breather which is localized in spatial and periodic