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Equivalent well-cell radius for hexagonal $\boldsymbol{K}$-orthogonal grids in numerical reservoir simulation

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ABSTRACT

Peaceman's equivalent well-cell radius for 2D Cartesian grids has been generalized to 2D uniform hexagonal $\boldsymbol{K}$-orthogonal grids in an anisotropic medium. An analytical expression for the equivalent well-cell radius for infinitely fine grids is derived. The derivation is based on a transformation to an isotropic image space and the solution of the pressure equation in this space.

1. Introduction

In reservoir simulation, the dimensions of a grid cell are generally much larger than the radius of a well. Therefore, the numerically computed pressure in the well cell will differ substantially from the bottomhole pressure in the well. For pressure-controlled wells, a formula for the difference between the well-cell pressure and the bottomhole pressure is needed. Peaceman [1,2] showed that for 2D simulations on uniform Cartesian grids, the pressure in the well cell occurs at a radius which is called the equivalent well-cell radius. For homogeneous anisotropic media he gave a formula for this radius and the difference between the well-cell pressure and the bottomhole pressure. Peaceman also gave an approximate method for calculating the equivalent well-cell radius, and this method was applied by [3–5] for simulations on hexagonal grids in isotropic media. Aavatsmark [6,7] extended Peaceman's formula to uniform hexagonal Voronoi grids in homogeneous isotropic media. Hexagonal grids have better rotational invariance than Cartesian grids, and they are therefore often preferred to reduce grid orientation effects. The most common use is for isotropic media, but in special cases, they may be applied for media with lateral anisotropy. An extension to anisotropic media is the topic of this paper.

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An alternative approach is obtained by applying a logarithmic discretization in the near-well zone [8]. This usually requires a logarithmic grid refinement in the vicinity of the well [9]. Excellent convergence is then obtained in the near-well zone, but the computational cost increases significantly, especially for multiphase flow [10]. Therefore, most commercial reservoir simulators apply Peaceman's approach with an equivalent well-cell radius to account for the singularity at the well [11,12]. In this paper, the alternative approach is not investigated.

2. Equations

In [6,7] it was shown that for uniform hexagonal Voronoi grids in a homogeneous isotropic 2D medium the equivalent well-cell radius is

$$r_0 = D\frac{e^{-\gamma}}{4}. \tag{1}$$

Here, D is the length of the diagonals in the hexagonal cells and $\gamma = 0.5772\ldots$ is Euler's constant. This formula holds for infinitely fine grids, but due to the slow variation with grid fineness, the formula may be applied for all grid sizes. In this paper, we discuss an extension of this formula to the case with an anisotropic medium. The pressure equation with a production well at the origin then reads

$$-\operatorname{div}\left(\frac{\boldsymbol{K}}{\mu}\operatorname{grad} p\right) = -\frac{Q}{h}\delta. \tag{2}$$

Here, p is the pressure, μ is the viscosity, Q is the production rate, h is the reservoir thickness and δ is Dirac's delta functional. The permeability tensor $\boldsymbol{K}$ is a constant, symmetric and positive definite matrix with different eigenvalues. The grid is uniform, hexagonal and $\boldsymbol{K}$-orthogonal. A grid is $\boldsymbol{K}$-orthogonal if the cell edges and the connecting lines between the centers of neighboring cells are $\boldsymbol{K}^{-1}$-orthogonal, see [13].

We can transform this problem to an isotropic image space with the area-preserving mapping

$$\boldsymbol{\xi} = (\det \boldsymbol{K})^{1/4}\boldsymbol{K}^{-1/2}\boldsymbol{x}. \tag{3}$$

Applying the transformation (3), Eq. (2) is simplified into the equation

$$-\frac{K}{\mu}\operatorname{div}\operatorname{grad} p = -\frac{Q}{h}\delta, \tag{4}$$

where $K = (\det \boldsymbol{K})^{1/2}$. If two vectors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ in the physical space are mapped into the vectors $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ in the image space, then

$$\boldsymbol{\xi}_1^{\mathrm{T}}\boldsymbol{\xi}_2 = (\det \boldsymbol{K})^{1/2}\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{K}^{-1/2}\boldsymbol{K}^{-1/2}\boldsymbol{x}_2 = (\det \boldsymbol{K})^{1/2}\boldsymbol{x}_1^{\mathrm{T}}\boldsymbol{K}^{-1}\boldsymbol{x}_2. \tag{5}$$

It follows that $\boldsymbol{K}^{-1}$-orthogonal lines in the physical space are mapped into orthogonal lines in the image space. Hence, the transformation (3) maps the $\boldsymbol{K}$-orthogonal grid in the anisotropic physical space into a Voronoi grid in the isotropic image space. Discretizing (2) on the hexagonal $\boldsymbol{K}$-orthogonal grid is completely equivalent to discretizing (4) on the hexagonal Voronoi grid. The discrete solutions are identical, and hence, the formula (1) for the equivalent well-cell radius in uniform hexagonal Voronoi grids applies. Letting $\boldsymbol{d}$ be the vector of one of the diagonals in the hexagonal cells in the uniform $\boldsymbol{K}$-orthogonal grid, it follows from (3) that

$$D = (\det \boldsymbol{K})^{1/4}\left\|\boldsymbol{K}^{-1/2}\boldsymbol{d}\right\|_2. \tag{6}$$

A peculiarity of formula (6) is that the length of the diagonals $\boldsymbol{d}$ in the hexagonal cells of the $\boldsymbol{K}$-orthogonal grid is not constant. However, the length of $\boldsymbol{K}^{-1/2}\boldsymbol{d}$ is constant. This is due to the fact that all corners of