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Endemic dynamics in a host–parasite epidemiological model within spatially heterogeneous environment

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ABSTRACT

In this paper, we investigate the endemic dynamics in a host–parasite model under combined frequency- and density-dependent transmission in a spatially heterogeneous environment. We give some properties of the parasite-free and parasite-driven extinction stationary solutions, and prove that the positive stationary solution set forms a bounded continuum which connects the parasite-free and parasite-driven extinction stationary solutions sets.

1. Introduction

In an effort to understand the host–parasite dynamics, Ryder et al. [1] established the following host–parasite epidemiological model in terms of the total host density (N) and the prevalence of infection (P):

$$\frac{dN}{dt} = N(b - hN) - \mu N - \alpha PN, \qquad \frac{dP}{dt} = P\Big(v(c + mN)(1 - P) - (b - hN) - \alpha(1 - P)\Big), \tag{1}$$

where $N(t) = S(t)+I(t)$ the density of the total host population, $P(t) = I(t)/N(t) \in [0, 1]$, and $S(t)$ and $I(t)$ denote the density of the uninfected (susceptible) and infected hosts at time t, respectively. And all parameters are positive, b and μ are the birth and natural death rates, respectively, α the rate of disease-induced mortality (i.e. virulence), h a coefficient of density-dependent host regulation. The transmission

$$Pv(c + mN)(1 - P) = \frac{v(c + mM)SI}{N} = \frac{(vm)SI}{N} + (vc)SI$$

is a function that combines two types of contacts: one is density-dependent $\frac{(vm)SI}{N}$ (also called mass action term), the other is frequency-dependent $(vc)SI$ (also called proportionate mixing term). Here, v is the per

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contact probability of transmission, m and c determine the amount of density- and frequency-dependent transmission, respectively.

Assume that susceptible and infected hosts move randomly [2–4], and the problem that we are attempting to address is: how do diffusion of hosts affect the parasite dynamics? For simplicity, assume that the diffusion coefficient of hosts is d, we therefore consider the following reaction–diffusion host–parasite model:

$$\begin{cases} \partial_t N - d\Delta N = N(b-hN) - \mu N - \alpha(x)PN, & x\in\Omega,\ t>0, \\ \partial_t P - d\Delta P = P\Big(v(c+mN)(1-P) - (b-hN) - \alpha(x)(1-P)\Big), & x\in\Omega,\ t>0, \\ \partial_{\mathbf{n}} N = \partial_{\mathbf{n}} P = 0, & x\in\partial\Omega,\ t>0, \\ N(x,0) = N_0(x)\geq 0, \qquad P(x,0) = P_0(x)\geq 0, & x\in\Omega, \end{cases} \tag{2}$$

where Δ the Laplace operator. The spatial heterogeneity is taken into account via the assumption that the rate of disease-induced mortality α is spatially dependent. Specifically, we require that $\alpha(\cdot)\in C^1(\bar{\Omega})$ and $\alpha(x)\neq 0$ for $x\in\bar{\Omega}$. It would be noted that the homogeneous case we mention in this paper is that α is spatially independent. And the habitat $\Omega\subset\mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$, $\mathbf{n}$ the outward unit normal vector on $\partial\Omega$. The homogeneous Neumann boundary condition implies that the system above is self-contained and there is no individual across the boundary. We require that the initial values $N_0(x), P_0(x)\in C(\bar{\Omega})$. And the steady state solutions of model (2) satisfy

$$\begin{cases} -d\Delta N = N(b-hN) - \mu N - \alpha(x)PN, & x\in\Omega, \\ -d\Delta P = P\Big(v(c+mN)(1-P) - (b-hN) - \alpha(x)(1-P)\Big), & x\in\Omega, \\ \partial_{\mathbf{n}} N = \partial_{\mathbf{n}} P = 0, & x\in\partial\Omega. \end{cases} \tag{3}$$

Throughout this paper, for $1\leq p\leq\infty$, let $L^p(\Omega)$ denote the Banach space of measurable functions u on Ω with the normal norms

$$\|u\|_p = \left(\int_\Omega |u(x)|^p\right)^{1/p}, \quad 1\leq p<\infty, \qquad \|u\|_\infty = \max_{\bar{\Omega}} |u(x)|.$$

Furthermore, let $\lambda_1(q)$ denotes the least eigenvalue of the problem

$$-d\Delta u + q(x)u = \lambda u, \quad x\in\Omega, \qquad \partial_{\mathbf{n}} u = 0, \quad x\in\partial\Omega,$$

here $q\in C(\bar{\Omega})$. It is well known that the mapping $q\to\lambda_1(q) : C(\bar{\Omega})\to\mathbb{R}$ is continuous and monotone increasing.

2. Main results

In this section, we will obtain sufficient conditions for the nonexistence and existence of positive solutions from the viewpoint of the bifurcation theory [5,6]. As a functional framework for the bifurcation theory, we introduce the following Banach spaces:

$$X := W_{\mathbf{n}}^{2,p}(\Omega)\times W_{\mathbf{n}}^{2,p}(\Omega), \qquad Y = L^p(\Omega)\times L^p(\Omega), \quad \text{for } p>n,$$

where $W_{\mathbf{n}}^{2,p}(\Omega) = \big\{w\in W^{2,p}(\Omega) | \partial_{\mathbf{n}} w = 0 \text{ on } \partial\Omega\big\}$. Then the Sobolev embedding theorem implies that $X\subset C^1(\bar{\Omega})\times C^1(\bar{\Omega})$ for $p>n$. It is easy to see that model (2) may have four stationary solutions:

(i) Extinction state $(0,0)$, which means that all parasites and hosts extinct;
(ii) Parasite-free stationary solution $(N_*,0)$, where $N_* = \frac{b-\mu}{h}$ for $b>\mu$.