Applied Mathematics Letters 61 (2016) 149–155

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

A stable numerical method for space fractional Landau–Lifshitz equations

Wei Yang[a], Dongling Wang[b,*], Lei Yang[c]

[a] *Hunan Key Laboratory for Computation and Simulation in Science and Engineering, Xiangtan University, China*

[b] *Department of Mathematics and Center for Nonlinear Studies, Northwest University, Xi'an, Shaanxi 710075, China*

[c] *Faculty of Information Technology, Macau University of Science and Technology, Macau*

ARTICLE INFO

Article history:
Received 5 May 2016
Received in revised form 26 May 2016
Accepted 26 May 2016
Available online 7 June 2016

Keywords:
Fractional Landau–Lifshitz equations
Micromagnetics
Gauss–Seidel projection
Split projection method

ABSTRACT

In this paper, we proposed a simple and unconditional stable time-split Gauss–Seidel projection (GSP) method for the space fractional Landau–Lifshitz (FLL) equations. Numerical results are presented to demonstrate the effectiveness and stability of this method.

1. Introduction

Micromagnetic modeling based on Landau–Lifshitz (LL) equations has played an increasingly important role in magnetic material design, as well as to characterize the magnetic behavior of different materials such as thin film heads, recording media, patterned magnetic elements, and magnetic nanowires. The evolution of magnetization in the ferromagnetic materials can be written as

$$M_t = -\gamma M \times H - \frac{\gamma\beta}{M_s} M \times (M \times H), \tag{1}$$

where $M_s = \|M\|$ is the saturation magnetization, and it is a constant far from the Curie temperature; the first and second terms on the right-hand side are called the gyromagnetic and the damping terms, respectively; γ is the gyromagnetic ratio, and β the dimensionless damping coefficient; H is the effective

* Corresponding author.
E-mail addresses: yangweixtu@126.com (W. Yang), wdymath@nwu.edu.cn (D. Wang), leiyang@must.edu.mo (L. Yang).

http://dx.doi.org/10.1016/j.aml.2016.05.014
0893-9659/

field, which can be computed from the Landau–Lifshitz free energy functional

$$H = -\frac{1}{M_s}\nabla \Phi\left(\frac{M}{M_s}\right) + \frac{A}{M_s^2}\Delta M + \mu_0 H_s + \mu_0 H_e$$

where $\Phi(\frac{M}{M_s})$ is the energy resulting from material anisotropy, $\frac{A}{M_s^2}|\nabla M|^2$ is the exchange interaction energy between the spins, H_s is called the stray field, H_e is the external field, and the parameters A and μ_0 are the exchange constant and permeability of vacuum, respectively. DeSimone et al. [1] derived the convergent magnetostatic energy through Fourier transforms which leads to a nonlocal space FLL equations. They obtained some important features of the ground-state magnetization pattern in the associated thin-film limit. Pu and Guo [2,3] considered the following FLL equations with periodic boundary conditions as

$$\begin{aligned} &m_t = m \times (-\Delta)^\alpha m + m \times (m \times (-\Delta)^\alpha m), \quad \text{in } \Omega \times [0, T], \\ &m(x,t) = u(x,t), \quad \text{on } \partial\Omega \times [0, T], \\ &m(x,0) = m_0(x,t), \quad \text{in } \Omega, \end{aligned} \tag{2}$$

where $(m, m) = 1$, and $(-\Delta)^\alpha$ is the fractional Laplacian defined as $(-\Delta)^\alpha m(x) = F^{-1}\left(|\xi|^{2\alpha} F(\xi)\right)$, where F is the Fourier transform. Guo et al. first proved the existence of smooth solutions for the fractional LL equations [2] and Pu et al. obtained the well-posedness of the fractional LL equations without Gilbert damping [3].

For the classical LL equations, i.e., $\alpha = 1$, many effective numerical methods are proposed [4–7]. Because of the strong nonlinearities and stiffnesses in the LL equations, the direct implicit numerical schemes of the system are not efficient and are difficultly applied in micromagnetics simulations. Wang et al. proposed a Gauss–Seidel projection (GSP) method for the LL equations [4]. Because of the effectiveness, simplicity and unconditionally stability, the GSP method has been widely accepted for micromagnetics simulations. To our best knowledge, there is no any result about the numerical method for the FLL equations (2). In this paper, an effective and simple semi-implicit scheme is obtained by extension of the GSP for classical LL equations [4] to the full FLL equations (2), and numerical results demonstrate that our algorithm is efficient and unconditionally stable for FLL equations.

The rest of the paper is organized as follows. In Section 2, the fractional central difference with second-order accuracy is employed to approximate the fractional Laplacian, then a split semi-implicit GSP scheme for the FLL equations are derived. In Section 3, numerical tests are provided to show the effectiveness and stability of the numerical method.

2. Semi-implicit split GSP scheme for the FLL equations

One of the main problem in numerical simulation for space fractional PDEs is to approximate the fractional Laplacian operator. The popular existed methods in literatures include that standard and shifted Grünwald formula, $L1$ and $L2$ approximations, and matrix transform method [8], and finite difference-quadrature approach [9], weighted and shifted method [10]. Another important way is the so-called fractional centered difference (F-CD) proposed in [11,12]. Ortigueira [11] introduced the F-CD by directly generalizing of the usual centered difference and it was shown to be equivalent the one-dimensional Riesz fractional derivative. Tarasov [12] studied some discrete systems with long-range interactions modeled by lattice fractional derivative, and the continuous limit give the medium equations with Riesz fractional derivative. Note that the lattice fractional derivative with some interaction kernel also leads to F-CD. Tarasov further considered three-dimensional lattice fractional vector calculus, and the differences and connections between the above two approaches of the introduction of F-CD are discussed in detail [13].