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A new method to find series solutions of a nonlinear wave equation

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ABSTRACT

We show that first-order approximate symmetries of a class of nonlinear wave equations contain Lie symmetries as particular cases. Then we present a new approach to find series solutions of the nonlinear wave equation which cannot be obtained by the standard Lie symmetry and approximate symmetry methods.
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1. Introduction

One of the fundamental applications of symmetry analysis for partial differential equations (PDEs) is to reduce the original PDEs into the differential equations which have fewer variables and are easier to be solved comparatively [1–5]. Meanwhile, as the extensive applications of symmetry method in various fields such as physics, chemistry, biology and engineering, the mathematical models described by PDEs constantly involve small parameters and admit fewer symmetries. Such a situation makes approximate symmetry method arise and then the studied models admit more affluent approximate symmetries [6–9]. In the rest of the letter, we will call Lie symmetry as exact symmetry in order to compare with approximate symmetry.

In the study of PDEs, exact symmetry generates exact similarity solutions while approximate symmetry gives approximate solutions. Then two related questions arise: among the admitted approximate symmetries, which one can give exact solutions of the PDEs? What is the relationship between the approximate symmetries generating exact solutions and the exact symmetries?

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Table 1
An exact symmetry classification of Eq. (1).

$F(u)$	Symmetry operators
Arbitrary	$X_1 = \partial_x,\ X_2 = \partial_t$
$e^{\lambda u}$	$X_1, X_2, X_3 = \frac{\lambda}{2} x\partial_x + \partial_u$
$u^\lambda (\lambda \neq -\frac{4}{3})$	$X_1, X_2, X_4 = \frac{\lambda}{2} x\partial_x + u\partial_u$
$u^{-4/3}$	$X_1, X_2, X_4, X_5 = -\frac{1}{3} x^2\partial_x + xu\partial_u$
u^{-2}	$X_1, X_2, X_4, X_6 = -e^{-\epsilon t}(\partial_t - \epsilon u\partial_u)$

The letter focus on the above two questions which are exemplified by a class of nonlinear wave equations

$$u_{tt} + \epsilon\, u_t = [F(u)u_x]_x, \tag{1}$$

where ϵ is a constant parameter and $F(u)$ is an arbitrary smooth function of u. Eq. (1) describes wave phenomena in shallow water, long radio engineering lines and isentropic motion of a fluid in a pipe, etc. [6,10]. Baikov et al. perform approximate symmetry analysis for Eq. (1) by expanding infinitesimal operator in the small parameter ϵ [6]. In [11], by expanding dependent variable u with respect to ϵ, we gave an infinite-order approximate symmetry classification and performed reductions with the optimal system.

For Eq. (1), by comparing exact symmetry with infinite-order approximate symmetry in [11], we find that first-order approximate symmetries contain exact symmetries as particular cases. Those infinite-order approximate symmetries whose first-order cases contain exact symmetries are used to construct new series solutions of Eq. (1). The main idea of the new method is to introduce arbitrary parameters in the infinitesimal operators and use such operators to construct new reduced equations, and then find explicit series solutions of the governing PDEs.

The rest of the letter is arranged as follows. In Section 2, the main results are given which include the connection between approximate symmetry and exact symmetry and the applications of new approach to Eq. (1). The last section contains a conclusion.

2. Main results

2.1. Connection between approximate symmetry and exact symmetry

We first consider the exact symmetry of Eq. (1). Viewing ϵ as a usual parameter and using standard Lie point symmetry method for Eq. (1), we obtain an exact symmetry classification listed in Table 1.

In what follows, we review an infinite-order approximate symmetry classification of Eq. (1) by the method originated from Fushchich and Shtelen [11].

Expanding the dependent variable u and functions $F(u)$ with respect to ϵ yields respectively

$$u = \sum_{k=0}^{\infty} \epsilon^k u_k, \qquad F(u) = F\left(\sum_{k=0}^{\infty} \epsilon^k u_k\right) = \sum_{k=0}^{\infty} \frac{\epsilon^k}{k!}\left[\frac{\partial^k F(u)}{\partial \epsilon^k}\right]_{\epsilon=0}.$$

Substituting the expansions into Eq. (1) and separating at each order of perturbation parameter ϵ, one has

$$u_{k,tt} + u_{k-1,t} = \left[\sum_{i=0}^{k} \frac{F^{(j)}(u_0)}{j_0! j_1! \cdots j_i!} u_{l_0}^{j_0} u_{l_1}^{j_1} \cdots u_{l_i}^{j_i} u_{k-i,x}\right]_x, \quad k = 0, 1, \ldots, \tag{2}$$

where, hereinafter, $u_{-1} = 0, j_0 + j_1 + \cdots + j_i = j, l_0 j_0 + l_1 j_1 + \cdots l_i j_i = i, 0 \le j \le i$ and $l_0, l_1, \ldots, l_i$ are not equal to zero and mutually inequivalent. The indexes $j_i, l_i, j, k (i, k, j = 0, 1, 2, \ldots)$ are nonnegative integers.

Then kth-order approximate symmetries of Eq. (1) correspond to the exact symmetries of system (2). The results are listed in Table 2.

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