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The zero limit of angular viscosity for the two-dimensional micropolar fluid equations

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ABSTRACT

In this paper, we consider the Cauchy problem of the incompressible micropolar fluids in dimension two. We prove that as the angular viscosity goes to zero (i.e., $\gamma \to 0$), the solution converges to a global solution of the original equations with zero angular viscosity. Convergence rates are also obtained.

1. Introduction

In this paper, we consider the following incompressible micropolar fluid equations in dimension two [1,2]:

$$u_t + u \cdot \nabla u + \nabla \pi = (\nu + \zeta)\Delta u + 2\zeta \nabla^{\perp} \omega, \tag{1.1}$$

$$\omega_t + u \cdot \nabla \omega = \gamma \Delta \omega - 2\zeta \nabla^{\perp} \cdot u - 4\zeta \omega, \tag{1.2}$$

$$\nabla \cdot u = 0, \tag{1.3}$$

where the unknown functions $u = (u_1, u_2)$, ω, and π, are the velocity field, micro-rotational velocity and pressure, respectively. The constants ν, γ and ζ are the Newtonian kinetic viscosity, the angular viscosity and the dynamic micro-rotation viscosity. Here, and in what follows,

$$\nabla^{\perp} = (\partial_y, -\partial_x), \qquad \nabla^{\perp} \cdot u = \partial_y u_1 - \partial_x u_2, \qquad \nabla^{\perp} \omega = (\partial_y \omega, -\partial_x \omega).$$

If $\omega = \text{Const}$ and $\zeta = 0$, then system (1.1)–(1.3) reduces to the classical Navier–Stokes equations which has been extensively studied [3–6].

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Due to its importance in mathematical physics, there have been many papers [7–13] concerning the existence, uniqueness and regularity problems of the micropolar fluid equations. By using the methods in [4,6], Galdi and Rionero [9] (see also [10]) proved the global existence of weak solutions of (1.1)–(1.3). The local existence and uniqueness of strong solutions to the micropolar flows was investigated in [12,13]. Dong and Zhang [11] showed the global regularity of the micropolar fluid equations with partial viscosity ($\gamma = 0$) in the 2D whole space. The zero limits of anger and micro-rotational viscosities (i.e., $\zeta, \gamma \to 0$) for the two-dimensional micropolar fluid equations with boundary effect were proved in [14]. In [15], they proved the vanishing microrotation viscosity limit ($\zeta \to 0$) in the case of zero kinematic viscosity ($\nu = 0$, $\gamma > 0$) or zero angular viscosity ($\nu > 0$, $\gamma = 0$). Some works about the partial viscosities can be found in [16–18].

Our main purpose in this paper is to justify the limit process. Formally, if $\gamma \to 0$, then system (1.1)–(1.3) becomes

$$\bar{u}_t + \bar{u} \cdot \nabla \bar{u} + \nabla \bar{\pi} = (\nu + \zeta)\Delta \bar{u} + 2\zeta \nabla^{\perp} \bar{\omega}, \tag{1.4}$$

$$\bar{\omega}_t + \bar{u} \cdot \nabla \bar{\omega} = -2\zeta \nabla^{\perp} \cdot \bar{u} - 4\zeta \bar{\omega}, \tag{1.5}$$

$$\nabla \cdot \bar{u} = 0. \tag{1.6}$$

Our result concerning the vanishing limit process from (1.1)–(1.3) to (1.4)–(1.6) can be formulated as follows.

Theorem 1.1. *Suppose that (u_0, ω_0) satisfy $u_0, \omega_0 \in H^m(\mathbb{R}^2)$ with $m \geq 3$, there exists a unique global solution (u, ω, π) to (1.1)–(1.3) on $\mathbb{R}^2 \times [0, T]$ satisfying*

$$u \in L^\infty(0, T; H^m(\mathbb{R}^2)) \cap L^2(0, T; H^{m+1}(\mathbb{R}^2)),$$
$$\omega \in L^\infty(0, T; H^m(\mathbb{R}^2)).$$

Moreover, there exists a positive constant C, independent of γ, such that

$$\sup_{0 \leq t \leq T} (\|u\|_{H^m}^2 + \|\omega\|_{H^m}^2) + \int_0^T (\|u\|_{H^{m+1}}^2 + \gamma \|\omega\|_{H^{m+1}}^2) dt \leq C(\|u_0\|_{H^m}, \|\omega_0\|_{H^m}, \nu, \zeta, T). \tag{1.7}$$

As a result, there exists a subsequence of (u, ω), still denoted by (u, ω), such that as $\gamma \to 0$,

$$\begin{cases} u \to \bar{u} & \text{strongly in } L^\infty(0, T; H^{m-1}), \\ \omega \to \bar{\omega} & \text{strongly in } L^\infty(0, T; H^{m-1}), \end{cases}$$

where $(\bar{u}, \bar{\omega})$ is a global strong solution of (1.4)–(1.6). In particular,

$$\sup_{0 \leq t \leq T} (\|u - \bar{u}\|_{H^{m-2}}^2 + \|\omega - \bar{\omega}\|_{H^{m-2}}^2) + \int_0^T \|u - \bar{u}\|_{H^{m-1}}^2 dt \leq C\gamma^2$$

and

$$\sup_{0 \leq t \leq T} (\|D^{m-1}(u - \bar{u})\|_{L^2}^2 + \|D^{m-1}(\omega - \bar{\omega})\|_{L^2}^2) \leq C\gamma.$$

2. Proof of Theorem 1.1

In [11], they used the Littlewood–Paley methods to prove the global regularity of (1.4)–(1.6). In fact, the term $\gamma \Delta \omega$ plays positive effect in the energy estimate. Therefore, by the same argument of [11], one can get the γ-independent estimates (1.7) for the solution of (1.1)–(1.3). Here, we will show the proof of (1.7) via a different approach.

Now, we introduce the following Kozono–Taniuchi's inequality [19] and the key estimate in [11].