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# Acoustic wave motions stabilized by boundary memory damping☆

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ABSTRACT

We consider the linear wave equation with acoustic boundary conditions (ABC) on a portion $\Gamma_1$ of the boundary and Dirichlet conditions on the rest of the boundary. The ABC contain a damping term of memory type with respect to the normal displacement of the point of $\Gamma_1$. Under some assumption on the memory kernel, we show that the associated operator matrix generates a strongly continuous semigroup of contractions on a Hilbert space, and the semigroup is strongly stable.


## 1. Introduction

Let $\Omega$ be a bounded domain in $\mathbb{R}^N$ with (sufficiently) smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$; $\Gamma_0$ and $\Gamma_1$ are closed nonempty disjoint subsets of $\Gamma$. The unit outer normal to $\Gamma_1$ is denoted by $n$. We consider the following system:

$$\begin{cases} \phi_{tt} = \Delta\phi, & \text{in } \mathbb{R}^+ \times \Omega, \\ \phi = 0, & \text{on } \mathbb{R}^+ \times \Gamma_0, \\ \partial_n \phi = \delta_t, & \text{on } \mathbb{R}^+ \times \Gamma_1, \\ -\phi_t = \delta + \int_0^t g(t-s)\delta_s(s)ds & \text{on } \mathbb{R}^+ \times \Gamma_1. \end{cases} \tag{1.1}$$

The system models sound wave propagation in a domain which is full of some kind of medium and with a portion of boundary made of light-weight viscoelastic material. The function $\phi(x,t)$ represents the velocity potential of the fluid, and $\delta(x,t)$ the normal displacement of the point $x \in \Gamma_1$ at time $t$. Each point of the surface $\Gamma_1$ acts like a viscoelastic element, which is characterized as a Maxwell model connected with an

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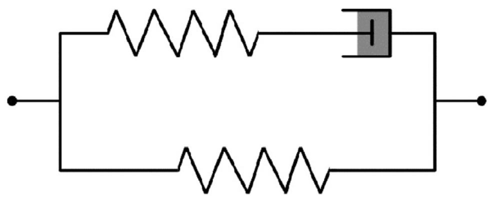

**Fig. 1.** Maxwell model connected with an elastic element.

elastic element (see Fig. 1), in response to the excess pressure. The points of $\Gamma_1$ do not influence one another; such a boundary is called to be *locally reacting* (see Morse and Ingard [1, p. 259]).

Starting from the work in [2], system (1.1) with the last boundary condition replaced by

$$-\rho_0\phi_t = m\delta_{tt} + d\delta_t + k\delta \quad \text{on } \mathbb{R}^+ \times \Gamma_1$$

($\rho_0 > 0$, and $m, d, k$ being nonnegative functions on $\Gamma_1$ with $m, k > 0$) has been studied by virtue of the semigroup theory (cf. [3,4] and the references therein). Recently, [5] investigated the stability of a viscoelastic wave equation with acoustic boundary conditions (see also [6]), where memory damping terms, w.r.t. the velocity potential $\phi$, appear both in the interior and boundary, besides the frictional boundary damping term $\delta_t$. In this paper, we will employ the semigroup approach to obtain wellposedness and strong stability of system (1.1) under some condition on the kernel $g$. For a related work on the wave equation

$$p_{tt} = \Delta p, \quad \text{in } \mathbb{R}^+ \times \Omega,$$

with the boundary condition

$$\partial_n p + \int_0^t g(t-s)p_s(s,\cdot)ds = 0, \quad \text{on } \mathbb{R}^+ \times \Gamma,$$

we refer to [7,8].

## 2. Preliminaries

We assume the following condition on the kernel $g$:

- there exists a right-continuous, nondecreasing, and nonconstant function $\lambda(\cdot)$ on $[0,\infty)$ satisfying $\lambda(0) = 0$, and

$$\int_0^\infty s^{-1} d\lambda(s) < \infty \tag{2.1}$$

such that

$$g(t) = \int_0^\infty e^{-st} d\lambda(s), \quad t > 0. \tag{2.2}$$

The condition means that $g$ is completely monotonic (cf. [9, p. 161, Theorem 12b]), and belongs to $L^1(0,\infty)$. A canonical example of those kernels satisfying the condition is $g(t) = pe^{-qt}$ with constants $p, q > 0$.

Using (2.2), we have

$$(g * \delta_t)(t,x) = \int_0^t g(t-s)\delta_s(s)ds = \int_0^\infty \psi(t,s,x)d\lambda(s), \quad \forall \delta \in C^1(\mathbb{R}^+;\ L^2(\Gamma_1;\mathbb{C}^n)),$$