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An alternative approach to compute wavelet connection coefficients

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ABSTRACT

In this paper we present a step by step algorithm to compute the wavelet connection coefficients using Daubechies wavelets. We address the treatment of scaling function derivatives which has a potential to simplify the computation of connection coefficients. We describe the evaluation of the integrals involving products of Daubechies wavelets and their derivatives. These connection coefficients are necessary for the wavelet-Galerkin approximation of differential or integral equations.

1. Introduction

Wavelets and the scaling functions associated with them have been increasingly used in both theoretical and applied research problems [1,2]. Wavelets are especially useful where the multiscale analysis is needed and the approximation changes depending on the chosen scale [3–6], specifically numerical solutions of ordinary and partial differential equations. Several methods can be used to compute these ODEs and PDEs such as Wavelet-Galerkin and Petrov–Galerkin [7–11]. For the Galerkin solution of differential equations, integrals of inner products of wavelets and their derivatives have to be computed. They cannot be computed analytically because of the limited regularity of the scaling functions and wavelets. Some of the studies that use Wavelet-Galerkin method in various kinds of research fields are as follows: Nadjafi et al. [12] used Wavelet-Galerkin method to solve the Volterra integral equations, Zhang and Cheng [13] used wavelet Galerkin formulation to optimize the acoustic enclosures, Hashish et al. [14] solved the 2-D heat equations using wavelet Galerkin method.

Wavelets have several useful properties that make them powerful tool to solve the ordinary and partial differential equations [15–17]. Daubechies wavelets [18,19] will be used in this paper due to their properties

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such as: (a) The basis functions of the Daubechies' wavelets are generated from the two functions called scaling function and mother wavelet with the use of translations and unitary scale transformations. (b) The functions are an orthonormal basis for square integrable functions with a compact support. (c) The scaling function is the solution of linear renormalization group equation. (d) The scaling function basis and wavelet basis can locally represent low-degree polynomials.

Several studies used different methods to compute connection coefficients [20–24]. Latto et al. [20] give the connection coefficients on unbounded intervals for the resolution $j = 0$ and Daubechies $K = 3$ wavelets. This approach can be successfully applied to problems in periodic nature or periodic distributions. To solve problems on bounded intervals Dianfeng et al. [23] used fictitious boundary approach to analyze SH wave but had some problems near resonance response. Chen et al. [10] used the wavelet Galerkin approximation to compute the connection coefficients on bounded interval for the resolution $j = 0$. All these methods involve some version of the method developed by Latto et al. [20] that uses scaling relation, normalization condition and moment conditions. The moment conditions are used at inhomogeneous part of the matrices that arise in these coefficients and their computation gets complicated with the higher resolution solutions. This paper describes an alternative way to compute the overlap integrals at bounded intervals with different resolutions. This paper is organized as follows: Section 2 describes the method to compute connection coefficients at bounded intervals with overlap integrals. Section 3 gives the numerical results and Section 4 gives the conclusion.

2. Computation of connection coefficients

In this section we give detailed information on how to compute connection coefficients. The integrals we will compute in this section are in the form:

$$\Gamma_n = \int f_1(x) f_2(x) \cdots f_n(x) dx$$

where the functions $f_i(x)$ are scaling functions, wavelets, derivatives of these functions or powers of x.

All of the functions that are used in the integrand of the coefficient Γ_n have simple transformation properties under scale transformation and translations. With the help of scaling equation and normalization condition we can reduce the computation of all of these quantities to finite linear algebra.

We use scaling equation $\phi(x) = \sum_{l=0}^{2K-1} h_l D T^l \phi(x)$ to derive following relations for the computations:

$$\int \phi_n^k(x) dx = \frac{1}{\sqrt{2^k}}$$
$$\frac{d}{dx} D = 2D \frac{d}{dx} \quad Dx = 2\sqrt{2} x D. \tag{1}$$

Also we use the scaling equations and definition of the derivatives of these equations:

$$\phi_m^k(x) = \sum_n H_{mn} \phi_n^{k+1}(x) \tag{2}$$
$$\phi_m^{k\prime} = 2 \sum_n H_{mn} \phi_n^{(k+1)\prime}.$$

We can use repeated application of these relations to increase k in each of the functions up to a resolution we need. Also the scale factor k of all functions in the integral can be increased or decreased by the same amount using the relation:

$$\int D^k f_1(x) D^k f_2(x) \cdots D^k f_m(x) dx = 2^{k(\frac{m}{2}-1)} \int f_1(x) f_2(x) \cdots f_m(x) dx, \tag{3}$$