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High accuracy analysis of nonconforming MFEM for constrained optimal control problems governed by Stokes equations

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ABSTRACT

In this paper, we propose a stable nonconforming mixed finite element method (MFEM) for the constrained optimal control problems (OCPs) governed by Stokes equations, in which the EQ_1^{rot}-constant scheme just satisfies the discrete inf–sup condition. The superclose and superconvergence results are obtained.

1. Introduction

Consider the following OCP with state constrained: find $(y, r, u) \in Y \times M \times U$, such that

$$\min_{u \in K^* \subset U} \left\{ g(y) + \frac{\alpha}{2} \int_\Omega (u - u_0)^2 dx \right\} \tag{1.1}$$

subject to

$$\begin{cases} -\Delta y + \nabla r = f + Bu, & \text{in } \Omega, \\ \text{div} y = 0, & \text{in } \Omega, \\ y = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

where α is a positive constant parameter, Ω is a bounded polygon in R^2, $x = (x_1, x_2) \in \Omega$, g is a convex functional, $f \in (L^2(\Omega))^2$ is a given function, B is a linear continuous operator, $Y = (H_0^1(\Omega) \cap H^3(\Omega))^2$,

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$M = L_0^2(\Omega) = \{r \in L^2(\Omega), \int_\Omega r dx = 0\}$, $U = (H^1(\Omega))^2$. In this paper, the Sobolev spaces and norms are both standard (see [1]). K^* is defined as

$$K^* = \{v \in U : v \leq 0, \text{ a.e. in } \Omega\}.$$

The above state constrained optimal control models play an important role in many science and engineering applications (cf. [2,3]). But the exact solution does not always exist or is difficult to be obtained, so researching the corresponding numerical algorithm becomes very meaningful. It is well known that the FEM is undoubtedly the most widely used numerical method in computing OCPs. In the recent decades, there appeared a lot of progresses on the FEMs of OCPs. [4] firstly proposed the conforming FEM for the OCPs and obtained the L^2-norm error estimate. Later on, [5] provided some important theoretical analysis and the so-called optimality conditions for OCPs. Based on this research, many studies have been devoted to the corresponding FEMs. For instance, [6] gained the convergence results of FE approximations to OCPs for semi-linear elliptic equations with finitely many state constraints. [7] extended these results to a less regular setting for the states, and also proved the convergence of FEMs for semi-linear distributed and boundary control problems. [8] investigated a discretization concept which utilizes for the discretization of the control variable the relation between adjoint state and control, in which they used the linear FEs to discretize the state equations and obtained the optimal error estimate in L^2-norm. On the other hand, some a posteriori error estimates of the FE approximations for OCPs governed by elliptic equations and Stokes equations were derived in [9] and [10], respectively. Recently, [11] gave a conforming MFEM for the OCPs governed by Stokes equations, which approximated the state and control variables by the bilinear-constant scheme, and deduced some superclose and superconvergence results for the control and state, respectively.

However, the works mentioned above are mainly contributions to the conforming FEMs. In fact, nonconforming FEMs have also attracted much more attentions of the engineers and scholars. Roughly speaking, nonconforming FEs have at least two advantages comparing with the conforming ones. The first aspect is that they are usually easier to be constructed to satisfy the celebrated discrete Babuska–Brezzi, or inf–sup stability condition, which is usually required in the MFEMs [1]. The second aspect comes from the domain decomposition methods point of view. For some Crouzeix–Raviart type elements with the degrees of freedom defined on the edges (or faces) of element, since the unknowns are associated with the element edges or faces, each degree of freedom belongs to at most two elements, the use of the nonconforming FEs facilitates the exchange of information across each subdomain and provides spectral radius estimates for the iterative domain decomposition operator [12]. In last thirty years, many scientists and engineers have successfully established the stable nonconforming FEM to some practical fluid and solid mechanics problems, linear or nonlinear Navier–Stokes problems, the elasticity related problems, and so on. In this paper, we construct a high accuracy MFEM for the constrained OCPs by employing the famous nonconforming EQ_1^{rot} element, which has been applied to many problems. For example, [13,14] studied its superconvergence properties for rectangular meshes, [15] considered its superconvergence behaviors for anisotropic meshes. Furthermore, this element was also employed to approximate the Maxwell's equations [16], nonlinear Sobolev equations [17] and some other different classic problems [18,19]. Recently, we also constructed some nonconforming FEM and MFEM for elliptic OCPs (see [20,21]). As far as we know, the nonconforming FEMs for bilinear optimal control problems have never been appeared. The rest of this paper is organized as follows. In the next section, we present the discrete formulation of (1.1)–(1.2) and some lemmas. In Section 3, we establish some superclose and superconvergence results.

2. The discrete formulation and some lemmas

By [22], we know that (1.1) has a unique solution (y, r, u) if and only if there is a co-state $(p, s) \in Y \times M$ such that (y, r, p, s, u) satisfies the following optimality conditions: