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Coexistence of activator and inhibitor for Brusselator diffusion system in chemical or biochemical reactions

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ABSTRACT

In this paper, we consider a reaction–diffusion system known as the Brusselator model with homogenous Neumann boundary conditions. The model is a system of two differential equations which describes a type of complex chemical or biochemical reaction system. We mainly study the coexistence of non-constant positive solutions to the model.

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1. Introduction

In general, most systems in chemical or biochemical reactions may be developed to model the morphogenesis and pattern formation. To study the dynamics in chemical or biochemical reaction–diffusion systems, it is important to master the kinetic generalities of the system near the critical points. These are the main contents of the chemical or biochemical reaction models, see, for example [1–8] and the references therein.

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with smooth boundary $\partial\Omega$. We consider a reaction–diffusion system: Brusselator reaction model, which is a trimolecular model that has been employed to describe chemical or biochemical morphogenesis and pattern formation [9]. The simplified and dimensionless form of Brusselator reaction model based on cubic reaction is as the following

$$\begin{cases} -\theta\Delta u = \lambda(A - (B+1)u + u^2 v), & x \in \Omega, \\ -\Delta v = \lambda(Bu - u^2 v), & x \in \Omega, \\ \partial_n u = \partial_n v = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

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where n is the outward unit normal vector on $\partial\Omega$ and $\partial_n = \partial/\partial n$. Variables u and v represent dimensionless chemical concentrations or densities of the activator and inhibitor in the reactor, respectively, and so are usually assumed to be non-negative. A, B are concentrations of prescribed constant reactants, θ is the diffusion coefficient of the concentration u, and λ is a measure of the size of the domain. So, A, B, θ, λ are always assumed to be positive constants. The reactor is isolated from the external environment, and no in- or out-fluxes of matter are allowed.

Unlike some other chemical or biochemical reactions (such as the Lengyel–Epstein model [1,8], which models the chlorite–iodide–malonic acid reaction), although the Brusselator model is not a model for any specific chemical system per se, this does not affect some critical analysis on its kinetics. Indeed, system (1.1) has received analytical or numerical studies as a model exhibiting Turing instability, such as [10–14] etc. The analysis of spatial–temporal organizing patterns was given in [10]. In [11], for one dimensional case, the authors used asymptotic expansions for u, v and λ to gain an expansion for the amplitude of positive non-constant solutions. The global behavior of the branches of nontrivial positive solutions was investigated in [12]. [13] gave some numerical results for periodic patterns. And in [14], the authors studied the priori estimate, non-existence and existence of positive non-constant steady states as the parameters varying.

Let $d_1 = d_1(\theta, \lambda) = \theta/\lambda, d_2 = d_2(\lambda) = 1/\lambda$. Then system (1.1) transforms into system

$$\begin{cases} -d_1\Delta u = A - (B+1)u + u^2 v, & x \in \Omega, \\ -d_2\Delta v = Bu - u^2 v, & x \in \Omega, \\ \partial_n u = \partial_n v = 0, & x \in \partial\Omega. \end{cases} \tag{1.2}$$

This work mainly aims at establishing the coexistence of activator and inhibitor in system (1.2). Our discussion is based on fixed point index results for compact operators in Banach space developed in [15]. As a note, the existence of positive non-constant solutions has been involved in [14], where, the analysis was mainly based on eigenvalues of the linearized operator of (1.1) at the unique constant solution. While, in the present paper, the existence result mainly depends on the direct analysis on the parameters A, B, θ and λ.

Let $\lambda_1 < \lambda_2 < \lambda_3 < \cdots$ be the sequence of eigenvalues for the elliptic operator $-\Delta$ subject to the homogenous Neumann boundary condition on Ω, where each $\lambda_i, i = 1, 2, 3, \ldots$, has multiplicity s_i. Let $\phi_{ij}, 1 \le j \le s_i$, be normalized eigenfunctions corresponding to λ_i. Then the set $\{\phi_{ij}\}, i \ge 1, 1 \le j \le s_i$, forms a complete orthogonal basis in $L^2(\Omega)$.

The main result of this work reads as follows.

Theorem 1.1. *Suppose that $B > 1$ and $\lambda_2 < (B-1)/d_1 < \lambda_3$. If the multiplicity of λ_2 is odd, then (1.2) has at least a non-constant positive solution for $d_2 > A^2(d_1\lambda_2 + 1)/\lambda_2((B-1) - d_1\lambda_2)$.*

2. Proof of the theorem

2.1. Notations and preliminaries

We first introduce some notations and preliminaries.

It is clear that system (1.2) has a unique constant solution $(u^*, v^*) = (A, BA^{-1})$. Let $\widetilde{u} = u - A, \widetilde{v} = v - BA^{-1}$. Then system (1.2) shifts into the form

$$-d_1\Delta\widetilde{u} = (B-1)\widetilde{u} + A^2\widetilde{v} + f_1(\widetilde{u}, \widetilde{v}), \quad x \in \Omega, \tag{2.1}$$

$$-d_2\Delta\widetilde{v} = -B\widetilde{u} - A^2\widetilde{v} + f_2(\widetilde{u}, \widetilde{v}), \quad x \in \Omega, \tag{2.2}$$

where f_1 and f_2 are higher order terms of $\widetilde{u}$ and $\widetilde{v}$ (In fact, $f_1(\widetilde{u}, \widetilde{v}) = BA^{-1}\widetilde{u}^2 + 2A\widetilde{u}\widetilde{v} + \widetilde{u}^2\widetilde{v}$, $f_2(\widetilde{u}, \widetilde{v}) = -f_1(\widetilde{u}, \widetilde{v})$.) Thus, the constant solution (u^*, v^*) transforms into $(0, 0)$.

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