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Mixed-type solitons and soliton interaction for the $(2+1)$-dimensional two-component long wave–short wave resonance interaction equations in a two-layer fluid through the Bell polynomials

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ABSTRACT

In this work, we investigate the $(2+1)$-dimensional two-component long wave–short wave resonance interaction equations in a two-layer fluid. Through the Bell polynomials, bilinear forms and mixed-type soliton solutions are derived. Such soliton phenomena as the V-type solitons, two parallel solitons with the periodic interaction, breather-type solitons and semi-breather-type solitons are observed, and the relevant parameter conditions are presented.

1. Introduction

Nowadays, theoretical and experimental studies on the solitons and relevant nonlinear evolution equations (NLEEs) have been carried out due to their attraction and application in fluids, plasmas, optical fibers and Bose–Einstein condensates [1–6]. Meanwhile, methods have been proposed to solve the NLEEs, e.g., the inverse scattering transformation, Bäcklund transformation, Darboux transformation, Hirota method and exponential function method [1,7–11].

In this work, through the Bell polynomials [12,13], we will analytically investigate the following $(2+1)$-dimensional two-component long wave–short wave resonance interaction equations in a two-layer

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fluid [14–18]:

$$i\left(S_t^{(1)}+S_y^{(1)}\right)-S_{xx}^{(1)}+LS^{(1)}=0, \tag{1a}$$

$$i\left(S_t^{(2)}+\sigma S_y^{(2)}\right)-S_{xx}^{(2)}+LS^{(2)}=0, \quad \sigma=\pm 1\,, \tag{1b}$$

$$L_t=2\left(|S^{(1)}|^2+|S^{(2)}|^2\right)_x, \tag{1c}$$

where the subscripts respectively denote the partial derivatives with respect to the scaled space coordinates x, y, and time t, $S^{(1)}$, $S^{(2)}$ and L respectively represent two short surface gravity packets and a long interfacial gravity wave. Eqs. (1) can be used to describe the resonance interaction between a long interfacial gravity wave and two short surface gravity packets propagating in a two-layer fluid [14–18]. Derivation for Eqs. (1) and some bright soliton solutions for Eqs. (1) with $\sigma=1$ or $\sigma=-1$ have been obtained [14–18]. Mixed-type soliton solutions for the multicomponent generalization of Eqs. (1) with $\sigma=1$ have been investigated [19].

However, to our knowledge, existing literature has not studied Eqs. (1) with the Bell polynomials, and mixed-type soliton solutions for Eqs. (1) with $\sigma=-1$. Moreover, some mixed-type interactions for Eqs. (1) have not been observed in the existing literature.

Therefore, in Section 2 of this work, through the Bell polynomials, bilinear form and mixed-type soliton solutions for Eqs. (1) will be derived. Besides, a set of the conditions for the mixed-type two-soliton solutions for Eqs. (1) with $\sigma=-1$ will be given. Section 3 will investigate the soliton interactions analytically and graphically, and give the relevant parameter conditions for such soliton phenomena as the V-type solitons, two parallel solitons propagation with the periodic interaction, breather-type solitons and semi-breather-type solitons. Section 4 will be our conclusions.

2. Bilinear form and soliton solutions

In this section, through the Bell polynomials, we will analytically obtain the bilinear form and mixed-type soliton solutions for Eqs. (1). Some concepts and formulas of the Bell polynomials are given in the Appendix.

2.1. Bilinear form via the Bell polynomials

Introducing the transformations

$$S^{(1)}=e^{V_1}, \qquad S^{(2)}=e^{V_2}, \qquad L=Q_{xx}, \tag{2}$$

where V_1 and V_2 are the complex differentiable functions with respect to x, y and t, and Q is a real one, and substituting them into Eqs. (1), we have

$$i\,(V_{1,t}+V_{1,y})-(V_{1,x}^2+V_{1,xx})+Q_{xx}=0, \tag{3a}$$

$$i\,(V_{2,t}+\sigma V_{2,y})-(V_{2,x}^2+V_{2,xx})+Q_{xx}=0, \tag{3b}$$

$$Q_{xt}+2\lambda-2\left(e^{V_1+V_1^*}+e^{V_2+V_2^*}\right)=0, \tag{3c}$$

with λ as a constant and $*$ as the complex conjugate. Via Expression (A.3), we need to set that

$$V_{j,xx}-Q_{xx}=W_{j,xx} \quad (j=1,2). \tag{4}$$

Thus, Eqs. (1) can be transformed into the following form:

$$i\,[\mathscr{Y}_t(V_1,W_1)+\mathscr{Y}_y(V_1,W_1)]-\mathscr{Y}_{2x}(V_1,W_1)=0, \tag{5a}$$

$$i\,[\mathscr{Y}_t(V_2,W_2)+\sigma\mathscr{Y}_y(V_2,W_2)]-\mathscr{Y}_{2x}(V_2,W_2)=0, \tag{5b}$$

$$V_{j,xt}-W_{j,xt}+2\lambda-2\left(e^{V_1+V_1^*}+e^{V_2+V_2^*}\right)=0. \tag{5c}$$